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ABSTRACT

Zero dimensional engine simulation models for gas turbines are used early in the design process as they do not require the component geometry details and experimentally obtained correlations that are required by 1D, 2D and 3D models. These 0D models require minimal data (like component characteristic maps) to carry out the on design, off design and transient simulations. The objective of the paper is to present two of map scaling techniques used to obtain component characteristic maps in detail. It also elaborates on engine simulation based on tracking the enthalpy property of the working fluid.

Key words: Gas Turbines Simulation, Enthalpy Based Formulation, Map Scaling.

INTRODUCTION

Simulation models for gas turbines are widely used in the design and development stage, control system design and find wide spread application in EHM (Engine Health Monitoring) systems that use a model based approach. The manner in which gas turbine simulation codes are used in the health monitoring system development is shown in **Figure 1**. First the simulation model is validated with healthy engine data (i.e. data obtained from the engine when the engine is without any deterioration or faults). Then the same ambient conditions and operating parameters such as throttle setting are given to the simulation model as that of the engine. If the engine is in a healthy state, the measurements from the engine

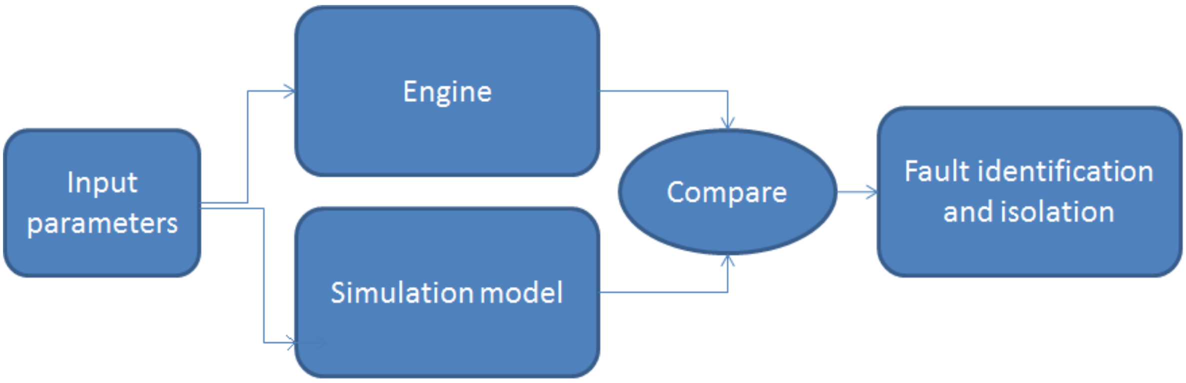


Figure 1: Use of simulation model in health monitoring

should match with the results of the simulation model at the corresponding locations. However, if the engine is in a deteriorated or in a faulty condition, differences arise between the simulation result and measurement. These differences are analysed by the comparator algorithm that passes its results to the fault identification and isolation algorithm. This algorithm not only detects the presence of the fault but also flags the component that is responsible for the deviations in measurements and lists probable faults in the component. This task is complicated as the signature to source relationship is often not unique and hence, multiple faults in the engine may have the same signature in the measurements. This makes it difficult to identify the precise component that is faulty. Increasing the number and quality of measurements greatly improves the reliability of the fault identification and isolation algorithms.

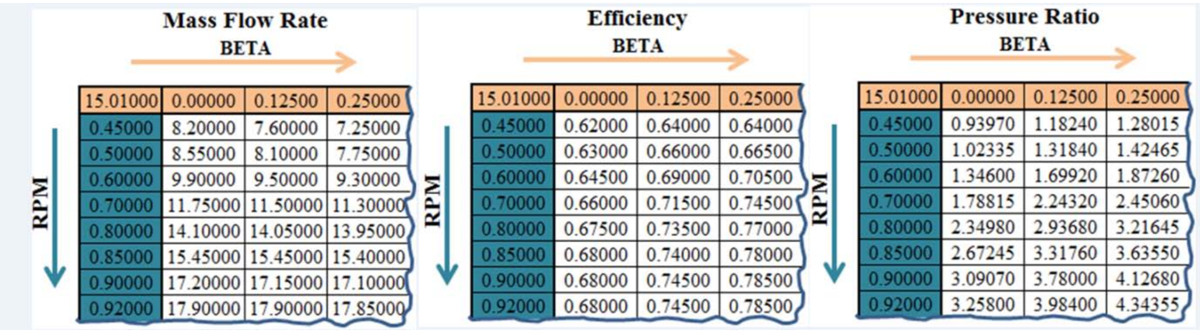
The importance of having a realistic simulation model is now apparent. But detailed two and three dimensional simulation models require a vast amount of component information and are also computationally expensive. One dimensional codes rely heavily on experimentally derived relations to account for the effects of the two un-modelled dimensions (typically radial and circumferential). Component geometry details of this level and experimental data are not available until the engine development program has progressed to an advanced stage. Typically, 0D models based on fundamental thermodynamic relations are used in the preliminary stages of design and EHM system development. These models rely on overall component parameters such as enclosed volume and spool inertia and do not require detailed geometric information of the components. As mentioned before, they make use of scaled component maps to simulate the off-design operation of the engine.

As mentioned before two of the important steps in model based fault identification are detection of presence of a fault and identification of the defective component. If possible, identifying the probable fault in the component would also be useful. Survey of techniques used in these phases is given in[1] and [2]. The use of gas turbine simulation models in the field of estimation techniques for estimating the parameters that are not normally measured from the engine is presented in [3]. Accounting for engine to engine variation that occurs during manufacturing is very important in building simulation models for EHM purposes as these manufacturing differences should not be identified as faults by the detection algorithm. Various means of tuning simulation models to specific engines exist and [4]describes a method of using a component level model with a fuzzy system to account for these engine to engine variations. Simulating the gas turbine engine with deterioration is very important to train the identification and isolation algorithms because these algorithms have to be trained on the relationship between various deteriorations and their effects on the measurements. Means of simulating the deteriorations such as blade fouling seal leakage and changes in the pattern factors of the combustors have been described in[5]. Process of applying weighted median filters to measurements from the engine such as gas temperature, fuel flow and rotor speed has been described in [6]. On the simulation model front, DYNGEN[7] was an early computer program for analysing the performance of gas turbines in transient and steady state mode. It combined the capabilities of the previous NASA codes and was written in FORTRAN. It used a modified Euler's method for solving the equations of the transient performance simulations. GTS (Gas Turbine Simulation) is yet another simulation program for turbofan engines capable of steady state and transient simulations. It is equipped with a GUI too and is described in [8]. An object oriented approach of gas turbine simulation is described in [9]. It concludes that the object oriented programming methodology will significantly improve the reliability, maintainability and manageability of the simulation code. Prediction of performance of individual components of the engine such as compressors and turbines is dealt with in [10]. One dimensional nonlinear simulation, accounting for inertia and volume dynamics of the components for a J85 turbojet engine has been described in [11]. Well written text books such as [12] and [13] give a very thorough explanation of the thermodynamics and principles of operation of the gas turbine engines. The CFD techniques and numerical solution methods used in developing full through simulation models and component models of gas turbine engines are described in [14]. This work focussed on the fluid flow simulation using Navier Stokes equation and not on thermodynamic analysis of the working fluid. The need of a representative simulation program in the development of an engine health monitoring program has been described in[15]. This work also describes the short comings of the conventional methods of condition monitoring such as vibration and oil quality monitoring.

Several well written text books provide detailed methodology of performing the 0D thermodynamic simulation of various configurations of engines based on constant and variable specific heats. Several commercial 0D engine simulation models are also available and they reveal very little about the equations being used. A detailed description of the map scaling techniques and engine simulation methodology using fundamental enthalpy polynomials to carry out the simulation has not been readily available. These methodologies are detailed in the following sections.

1. MAP SCALING TECHNIQUES

Component characteristic maps are essential in the off design simulation of components. The map scaling methods for different components are explained using the compressor characteristic map. Each point on the compressor characteristic map represents a state of the compressor at which it has a unique combination of mass flow rate, efficiency and pressure ratio. The operating point of the compressor on the compressor characteristic map is specified by the two independent variables β and RPM. Using these two parameters as independent variable, the compressor map is actually a set of three maps as shown below.


Mass Flow Rate

BETA

15.01000	0.00000	0.12500	0.25000
0.45000	8.20000	7.60000	7.25000
0.50000	8.55000	8.10000	7.75000
0.60000	9.90000	9.50000	9.30000
0.70000	11.75000	11.50000	11.30000
0.80000	14.10000	14.05000	13.95000
0.85000	15.45000	15.45000	15.40000
0.90000	17.20000	17.15000	17.10000
0.92000	17.90000	17.90000	17.85000

RPM

Efficiency

BETA

15.01000	0.00000	0.12500	0.25000
0.45000	0.62000	0.64000	0.64000
0.50000	0.63000	0.66000	0.66500
0.60000	0.64500	0.69000	0.70500
0.70000	0.66000	0.71500	0.74500
0.80000	0.67500	0.73500	0.77000
0.85000	0.68000	0.74000	0.78000
0.90000	0.68000	0.74500	0.78500
0.92000	0.68000	0.74500	0.78500

RPM

Pressure Ratio

BETA

15.01000	0.00000	0.12500	0.25000
0.45000	0.93970	1.18240	1.28015
0.50000	1.02335	1.31840	1.42465
0.60000	1.34600	1.69920	1.87260
0.70000	1.78815	2.24320	2.45060
0.80000	2.34980	2.93680	3.21645
0.85000	2.67245	3.31760	3.63550
0.90000	3.09070	3.78000	4.12680
0.92000	3.25800	3.98400	4.34355

RPM

Figure 2: Format of the three compressor maps, giving mass flow rate, efficiency and pressure ratio

These component maps are generated through extensive experimental testing of the component on the test rig or detailed 3D CFD analysis of the blade profiles along with enclosed volumes. For engines under development, the information to do these 3D simulations (such as blade profile and enclosed volume geometry details) is not available in the preliminary stages for engines under development. Even for proven engines in service, this information is rarely available as it is of great value to engine manufacturers and is rarely published. Thus map scaling procedures have to be resorted to obtain component maps from similar engines. This section details two such map scaling methods.

A) Single point scaling method

The first method [12] is to scale the entire map using the scaling factors that were obtained by taking ratios of the design point value of the component and the design point value of the reference map being simulated. The mass flow rates and RPM mentioned in the following formulas are corrected quantities without accounting for real gas effects.

$$\dot{m}_{scaleFactor} = \left(\frac{\dot{m}_{refMap}}{\dot{m}_{targetEngine}}\right)^{-1} \tag{1}$$

On obtaining the scale factor, the entire compressor map that gives the mass flow rate is multiplied by this scale factor. Similarly, other two scale factors are also computed and the reference pressure ratio and efficiency maps are multiplied at all points by these scale factors.

$$Pr_{scaleFactor} = \left(\frac{Pr_{refMap}-1}{Pr_{targetEngine}-1}\right)^{-1} \tag{2}$$

$$\eta_{scaleFactor} = \left(\frac{\eta_{refMap}}{\eta_{targetEngine}}\right)^{-1} \tag{3}$$

Obviously, the scaled map and the design point of the target engine will match exactly at the design point of the target engine. Engine manufacturers who have access to a large number experimentally generated maps, chose a reference map which has similar mass flow rate and performance characteristics as the component being analysed. In the absence of a similar component map for scaling, we are generally forced to scale the map of the closest component that we can find. The design points of the two maps that were chosen as the reference map and the target map are given below in ***Table 1***. The reference map is a compressor map provided as a sample map along with the commercially available GSP simulation program. The target map is provided along with a NASA report on thermodynamic performance analysis of aircraft gas turbines [16].

Table 1: Design point values of reference and target components.

Parameter	Reference map	Target map
Mass flow rate (Kg/sec)	19.87	206
Efficiency	0.87	0.852
RPM	1	1
Pressure ratio	6.6292	23

In order to demonstrate the single point map scaling method, we consider these two compressor maps, scale the first map (reference map) to the design point of the second map and compare it with the actual second map(target map).The resulting plots are shown in **Figure 2** to **Figure 4**.

B) Applying factors and deltas to a map

The second map scaling method of applying factors and deltas to a reference map can be used only if data is available at more than two operating points for the target engine. The equations that have to be solved to obtain the scale factors and deltas are,

$$\dot{m}_{targetEngine} = F1 * \dot{m}_{refMap} + \Delta 1$$
$$\eta_{targetEngine} = F2 * \eta_{refMap} + \Delta 2$$
$$PR_{targetEngine} = F3 * PR_{refMap} + \Delta 3 \tag{4}$$

The dependent parameters of our target engine are related to the dependent parameters of the reference engine through these scale factors and deltas. When we have operating point data at more than two points for our target engine, the factors and deltas are evaluated from an over determined system of equations. Contrary to the single point method, this method gives us scaled maps that do not match with the target engine data even at design point. This is because the scaling is done through both multiplicative scale factors and additive deltas. These factors and deltas are obtained by solving the system of equations in a least square error sense and hence do not exactly match the target engine's operating point at any point. The advantage is that the error between the different operating point data and the scaled map will be minimised in a least square sense.

The important question that arises is the problem finding points in the reference map to which the operating point data of target engine can be equated to. This step is required to form the system of equations. These corresponding points are found by first normalising the above operating point data and finding data points on the reference map that have the same normalised values. As the number of operating points considered for finding the factors increase, the scaling becomes better in a least square sense over a larger region in the map. The plots shown in **Figure 6** and **Figure 7**show that the scaled map better represents the target engine map when more number of points is used in the evaluation of factors and deltas. **Figure 8**and **Figure 9** compare the scaled and target maps of mass flow rate and pressure ratio.

Before we apply the formula to find the factors and deltas, since we have both the target and the reference map, it is instructive to see the variation of pressure over the speed range of both the compressors as shown in **Figure 5**. The reference map is first normalised and the normalised target map is plotted over it to see this variation. From the figure it can be seen that the decrease in pressure ratio of the target engine at lower RPM is much higher than the decrease in pressure of the reference engine. Only those data points that fall within the upper and lower limits can be utilised, as the other data points will not have an equivalent normalised data point in the reference map.

2. PERFORMANCE SIMULATION

Engine performance simulation is an essential part of an engine health monitoring module. The EHM module typically uses two approaches. The first is to run an engine simulation model with the same inputs as that of a real engine and compare the measurements with simulated values. The fault identification algorithms works on the comparative data to identify the faulty components. In data based EHM module too, the simulation models are used to train the neural network models that simulate the engine. Training the neural networks is more practical with simulated data rather than real data as data with faults is not readily available whereas any fault can be simulated in a mathematical model and data can be generated.

C) **On design Simulation**

On design simulation or design point simulation is one in which the engine components are running at their respective design points. The component characteristic map is not required for on design simulation. Any changes from the design point values such as change in mass flow rate or fuel flow rate results in the operating point shifting away from the design point. All other points on the characteristic map are off design points and an iterative process has to be carried out to simulate the engine at these operating points. A detail description of the simulation model is given in the following sections.

1) Air model:

The air is modelled as a mixture of several gases in enthalpy based formulations. This is done so that the change in composition of gases after combustion can be taken into account after the combustor. The variation of gas properties with respect to temperature is given as a polynomial function of the following form.

$$\frac{C_p}{R} = a_1 * T^{-2} + a_2 * T^{-1} + a_3 + a_4 * T + a_5 * T^2 + a_6 * T^3 + a_7 * T^4 \qquad (5)$$

Each of these gases has its own variation of gas properties with respect to temperature. For example, for Nitrogen, the constants $a_1, a_2, .. a_7$ are given in Table 1.

Table 2 : Values of constants for Nitrogen

a_1	2.21E+04
a_2	-3.82E+02
a_3	6.08E+00
a_4	-8.53E-03
a_5	1.38E-05
a_6	-9.63E-09
a_7	2.52E-12
b_1	7.11E+02
b_2	-1.08E+01

The constants b_1 and b_2 are constants of integration and are used when expressing the enthalpies of the gases as a function of temperature. From the above constants, the variation of specific heat of Nitrogen with temperature can be obtained. Values of these constants for a large number of gases can be obtained from the “thermo.inp” file provided with the NASA CEA program. While using constants from the NASA CEA program, it is important to note the temperature range for which these constants are valid. Since the constants are provided for two temperature ranges, it is usually convenient to use an “if loop” to choose the constants based on the temperature of the air.

While simulating air from inlet up to the combustor entry, we can model air as a mixture of individual gases or use the constants provided directly for air. While modelling air as a mixture of gases it is necessary to take molar average of properties and the certain composition of air has to be assumed for that purpose. The mole fraction of gases in air as used in the NASA CEA programs are Nitrogen = 78.084%, Oxygen = 20.9476%, Argon = 0.9365%, Carbon di Oxide = 0.0319%.

2) Sequence of simulation:

One of the major differences between the physical flow of air in the engine and the methodology of simulation is that the compressor is simulated before the inlet in a simulation model [12]. The compressor is an active component that draws air into the engine and determines the mass flow rate. The beta value and the RPM of the compressor at design point are fixed and uniquely specify the operating pressure ratio, corrected mass flow rate and efficiency at the design point. Once the corrected mass flow rate is obtained from the compressor map and total pressure at compressor entry is obtained from inlet model, the actual mass flow rate can be obtained using the following equation.

$$\dot{m}_{actual} = \dot{m}_{corrected} * \frac{\left(\frac{P}{P_{seaLevel}}\right)}{\sqrt{\frac{T}{T_{seaLevel}}}} \tag{6}$$

The total temperature at compressor inlet will be same as that of inlet total pressure entry as inlet is a passive component and no energy extraction or supply takes place.

3) Inlet:

From the engine modelling point of view, the effect of a sub sonic inlet[1]is its contribution to the total pressure loss of the working fluid. This total pressure loss is expressed in terms of ram recovery factor and is mainly a function of the inlet flow Mach number. This factor also includes the pressure loss that occurs in the free stream, before the flow reaches the leading edge of the intake.

$$Ram\ recovery\ factor = \frac{P_1 - P_\infty}{P_0 - P_\infty} \tag{7}$$

The MIL E 5008-B standard[17] gives expressions that can be used to express the inlet ram recovery factor in terms of the Mach number.

An alternative is to rely on an inlet map generated through experimental wind tunnel data. This requires the exact profile of the inlet to be known and is usually not feasible in preliminary model development stages.

4) Compressor:

The main job of the compressor model is to obtain the operating pressure ratio, efficiency and mass flow rate from the beta value and RPM using the compressor map. The compressor map can be expressed in the form of three 3D look up tables. All three lookup tables have beta and RPM as their independents and efficiency, mass flow rate and pressure ration as dependents respectively.

Once the 3 dependents are obtained, the exit temperature of the gases from the compressor has to be calculated while accounting for compressor isentropic efficiency. This can be done via the following procedure.

1. Evaluate $\int_{T_{ref}}^{T_2} \frac{C_p}{RT} dt$ from a reference temperature (298.15K). To evaluate the above integral, use the expression for Cp in terms of temperature.
2. Try different compressor exit temperatures (T_{3isen}) in the following equation until the equation is satisfied.

[1]For supersonic intakes, additional shock losses have to be accounted for.

$$\int_{T_{ref}}^{T_{3isen}} \frac{C_p}{RT} dt = \int_{T_{ref}}^{T_2} \frac{C_p}{RT} dt + \log\left(\frac{P_3}{P_2}\right) \tag{8}$$

3. Once the isentropic temperature (T_{3isen}) for which the above equation is satisfied is obtained, the enthalpy corresponding to that temperature has to be calculated.
4. Isentropic change in enthalpy can be calculated from the above enthalpy values and the actual enthalpy change can be obtained by using the compressor isentropic efficiency.

$$\Delta h_{isentropic} = h(T_{3isen}) - h(T_2) \tag{9}$$

$$\Delta h_{actual} = \frac{\Delta h_{isentropic}}{\eta_{compressor}} \tag{10}$$

$$h_{3actual} = h_2 + \Delta h_{actual} \tag{11}$$

5. Once the actual enthalpy at exit has been calculated, different temperatures (T_3) can be tried until the enthalpy for the trial temperature matches with the enthalpy at exit.

$$h(T_3) = h_{3actual} \tag{12}$$

6. If cross section area is given, evaluate the static flow properties.

5) Combustor:

For on design simulation, we need a combustor model that will provide a reasonable estimate of exhaust gas composition along with exit temperature of the gases. NASA CEA[18] code can be used to our advantage here. This code solves for equilibrium concentration of the different reacting species and their products by minimising the Gibbs free energy function. The executable file of the NASA CEA code is distributed by NASA along with sample example files and full input files. The major inputs the NASA code expects for JetA (1) fuel combustion simulation in "hp" mode are

- Gas temperature at inlet to the combustor
- Fuel temperature at inlet to the combustor
- Pressure of gas at to the combustor
- Air fuel ratio

These values can be written to a text file once the simulation has been completed up to the compressor stage. This file has to be appropriately formatted and supplied as input to the NASA CEA code. The code in turn writes an output file detailing the output gas composition and the temperature of the exit gas. The code however, assumes constant pressure combustion and does not provide any information about the pressure drop in the combustor. To that end, we have to use the combustor pressure loss factor obtained from similar combustor designs.

$$PRcombustor = (1 - combustor\ pressure\ loss\ factor) \tag{13}$$

$$Combustor\ exit\ pressure = Combustor\ inlet\ pressure * PRcombustor \tag{14}$$

6) *Turbine:*

The turbine design point simulation is the inverse of the compressor simulation. The first thing to do in the turbine model is to alter the standard gas model according to the exhaust gas composition supplied by the NASA CEA code.

$$C_{p_{products}}(T) = C_{p_{Ar}}(T) * Ar_{mF} + C_{p_{co_2}}(T) * CO2_{mF} + C_{p_{H_2O}}(T) * H2O_{mF} + C_{p_{NO}}(T) * NO_{mF} + C_{p_{NO_2}}(T) * NO2_{mF} + C_{p_{N_2}}(T) * N2_{mF} + C_{p_{O_2}}(T) * O2_{mF} \tag{15}$$

In the above equation, Ar_{mF} denotes the mole fraction of Argon in the mixture, $C_{p_{Ar}}(T)$ denotes the specific heat of Argon at the temperature T. The operating point of the turbine in the turbine characteristics map is uniquely determined by beta and RPM. The operating point is chosen such that it satisfies two major conditions. . The two conditions that must be satisfied for this design point performance simulation are mass flow balance and energy balance. At steady state, the practice is to fix the operating point on the map using energy balance and then check whether the mass flow balance has been satisfied. If the turbine and compressor has been properly matched, the error should ideally be zero. The sequence of calculations to be done is shown below.

1. From turbine inlet pressure, temperature, turbine efficiency and mechanical efficiency of the transmission system, calculate the exit temperature and pressure of the turbine. In order to this we need to get the inlet enthalpy to the turbine. The work required by the compressor after accounting for mechanical transmission losses due to in-efficiencies have to be subtracted and exit enthalpy of the turbine is obtained. Then a temperature corresponding to this enthalpy has to be found iteratively.

$$H_{turbineExit} = H_{turbineEntry} - \frac{\Delta H}{\eta_{mechTransmissionEfficiency}} \tag{16}$$

2. This temperature drop would have resulted in a certain pressure drop. But due to the isentropic efficiency not being 1, the actual pressure drop will be more than the bare minimum pressure drop in the turbine. Thus, actual pressure drop has to be calculated by accounting for isentropic efficiency of the turbine.

$$T_{isentropic} = T_{in} - \frac{T_{in} - T_{actual}}{\eta_{turbine}} \tag{17}$$

$$\int_{T_{ref}}^{T_{turbineInlet}} \frac{C_p}{RT} dt - \int_{T_{ref}}^{T_{turbineExitIsen}} \frac{C_p}{RT} dt = \log\left(\frac{P_{turbineExit}}{P_{turbineInlet}}\right) \tag{18}$$

3. The RPM of the system is already fixed. From the RPM and pressure ratio, find the beta at which turbine is operating from the lookup tables.
4. For the obtained RPM and beta, find the mass flow rate that can be ingested from the mass flow rate maps. If the turbine and compressor had been properly matched, this mass flow will be the same as the one that is being pushed into the turbine.
5. If cross section area is given, evaluate the static flow properties.

7) *Nozzle:*

The nozzle is relatively straight forward from a simulation point of view. No energy is given or taken from the fluid and the only consideration is to check whether the flow is choked or not while calculating the thrust. The sequence of calculations to be done are

1. Calculate the specific heat and other gas properties from the total temperature.
2. Check whether the nozzle is choked (based on the exit pressure ratio) and evaluate the thrust accordingly.
3. Once static properties are obtained at nozzle exit, calculate the gas properties again for the static temperature. If they differ significantly from gas properties obtained from total temperatures, iterate until convergence is achieved.
4. As a check, calculate the mass flow rate and see if it matches the mass flow rate being fed into the nozzle.

The results of the simulation program (gasTurbNal), developed in house, are compared with commercial gas turbine simulation software for a single spool turbo jet engine configuration. The design point of the turbojet being simulated is given in ***Table 1*** and the percentage errors of the comparison are shown in **Figure 10**

D) Off design simulation

Off design simulation, as told before, is an iterative process and makes use of the on design simulation model extensively. For a single spool turbojet, the model contains four independents and four dependents. The independents are

- $\dot{m}_{inlet}$(mass flow rate of air into the inlet)
- $combustor\ exit\ temperature(COT)$
- $\dot{w}_f(fuel\ flow\ rate\ into\ the\ combustor)$
- Operating point (β) of the compressor

The dependents or the error variables are

- $\dot{m}_{error}$ at turbine
- $\dot{m}_{error}$ at compressor
- $\dot{m}_{error}$ at nozzle
- Combustor outlet temperature error.

Our job is to find a combination of independents such that the dependents are made zero. For example, the combustor outlet temperature is a guessed independent. Its value is guessed such that the error in mass flow rate at turbine is zero. If the $\dot{w}_f$(fuel flow rate) that we have guessed matches the combustor exit temperature, the combustor exit temperature is driven to zero. The Jacobian matrix based iterative scheme presented here is used to make these successive guesses such that errors are driven to zero.

1. First thing to do is to modify the on design code such that it returns the four error terms after accepting the four independents as input.

- $\dot{m}_{error}$ at turbine : error between the combustor exit mass flow rate and the turbine mass flow rate from the map
- $\dot{m}_{error}$ at compressor: error between the inlet exit mass flow rate and the compressor mass flow rate from the map

- $\dot{m}_{error}$ at nozzle: error between the turbine exit mass flow rate and the mass flow rate corresponding to the static properties at nozzle exit.
- Combustor outlet temperature error: error between the guessed COT and the combustor exit temperature for guessed flow rate of fuel.

2. Evaluate the partial derivative matrix of these error terms with respect to the guessed independents. The on design code can be used for this evaluation in the following manner. Each of the elements of the guess vectors can be perturbed by 0.1 % to the left and to the right to obtain the variation of four error terms with respect to that error term. Some of the errors may be impervious changes in certain guess terms. For example, compressor mass flow rate error is not affected by changes in the weight flow rate of fuel. These partial derivatives may turn out to be zero. The structure of the partial derivative matrix is as shown below:

$$Jacobian = \begin{pmatrix} \frac{\partial \dot{m}_{errorComp}}{\partial \beta} & \frac{\partial \dot{m}_{errorComp}}{\partial COT} & \frac{\partial \dot{m}_{errorComp}}{\partial \dot{w}_f} & \frac{\partial \dot{m}_{errorComp}}{\partial \dot{m}_{inlet}} \\ \frac{\partial \dot{m}_{errorTurbine}}{\partial \beta} & \frac{\partial \dot{m}_{errorTurbine}}{\partial COT} & \frac{\partial \dot{m}_{errorTurbine}}{\partial \dot{w}_f} & \frac{\partial \dot{m}_{errorTurbine}}{\partial \dot{m}_{inlet}} \\ \frac{\partial \dot{m}_{errorNozzle}}{\partial \beta} & \frac{\partial \dot{m}_{errorNozzle}}{\partial COT} & \frac{\partial \dot{m}_{errorNozzle}}{\partial \dot{w}_f} & \frac{\partial \dot{m}_{errorNozzle}}{\partial \dot{m}_{inlet}} \\ \frac{\partial COT_{error}}{\partial \beta} & \frac{\partial COT_{error}}{\partial COT} & \frac{\partial COT_{error}}{\partial \dot{w}_f} & \frac{\partial COT_{error}}{\partial \dot{m}_{inlet}} \end{pmatrix} \tag{19}$$

3. Once the Jacobian has been evaluated, estimate the error vector at the current operating point.
4. With the Jacobian and the error vectors evaluated, we can compute the guess vector for the next iteration by solving the following equation. In the following, the errors are expressed as a function of the guess vectors and Newton Raphson's method is used to obtain the next guess vector.

$$f\begin{pmatrix} \beta \\ COT \\ \dot{w}_f \\ \dot{m}_{inlet} \end{pmatrix} = \begin{pmatrix} \dot{m}_{errorComp} \\ \dot{m}_{errorTurbine} \\ \dot{m}_{errorNozzle} \\ COT_{error} \end{pmatrix} \Rightarrow 0 \tag{20}$$

$$f\begin{pmatrix} \beta \\ COT \\ \dot{w}_f \\ \dot{m}_{inlet} \end{pmatrix} + \begin{pmatrix} \frac{\partial \dot{m}_{errorComp}}{\partial \beta} & \frac{\partial \dot{m}_{errorComp}}{\partial COT} & \frac{\partial \dot{m}_{errorComp}}{\partial \dot{w}_f} & \frac{\partial \dot{m}_{errorComp}}{\partial \dot{m}_{inlet}} \\ \frac{\partial \dot{m}_{errorTurbine}}{\partial \beta} & \frac{\partial \dot{m}_{errorTurbine}}{\partial COT} & \frac{\partial \dot{m}_{errorTurbine}}{\partial \dot{w}_f} & \frac{\partial \dot{m}_{errorTurbine}}{\partial \dot{m}_{inlet}} \\ \frac{\partial \dot{m}_{errorNozzle}}{\partial \beta} & \frac{\partial \dot{m}_{errorNozzle}}{\partial COT} & \frac{\partial \dot{m}_{errorNozzle}}{\partial \dot{w}_f} & \frac{\partial \dot{m}_{errorNozzle}}{\partial \dot{m}_{inlet}} \\ \frac{\partial COT_{error}}{\partial \beta} & \frac{\partial COT_{error}}{\partial COT} & \frac{\partial COT_{error}}{\partial \dot{w}_f} & \frac{\partial COT_{error}}{\partial \dot{m}_{inlet}} \end{pmatrix} \begin{pmatrix} \Delta\beta \\ \Delta COT \\ \Delta\dot{w}_f \\ \Delta\dot{m}_{inlet} \end{pmatrix} = 0 \tag{21}$$

On solving the above system of equations, we obtain the change in guess vector that can be added to the previous guess vector to obtain the guess vector for the next iteration.

$$\begin{pmatrix} \Delta\beta \\ \Delta COT \\ \Delta\dot{w}_f \\ \Delta\dot{m}_{inlet} \end{pmatrix} = \begin{pmatrix} \frac{\partial \dot{m}_{errorComp}}{\partial\beta} & \frac{\partial \dot{m}_{errorComp}}{\partial COT} & \frac{\partial \dot{m}_{errorComp}}{\partial \dot{w}_f} & \frac{\partial \dot{m}_{errorComp}}{\partial \dot{m}_{inlet}} \\ \frac{\partial \dot{m}_{errorTurbine}}{\partial\beta} & \frac{\partial \dot{m}_{errorTurbine}}{\partial COT} & \frac{\partial \dot{m}_{errorTurbine}}{\partial \dot{w}_f} & \frac{\partial \dot{m}_{errorTurbine}}{\partial \dot{m}_{inlet}} \\ \frac{\partial \dot{m}_{errorNozzle}}{\partial\beta} & \frac{\partial \dot{m}_{errorNozzle}}{\partial COT} & \frac{\partial \dot{m}_{errorNozzle}}{\partial \dot{w}_f} & \frac{\partial \dot{m}_{errorNozzle}}{\partial \dot{m}_{inlet}} \\ \frac{\partial COT_{error}}{\partial\beta} & \frac{\partial COT_{error}}{\partial COT} & \frac{\partial COT_{error}}{\partial \dot{w}_f} & \frac{\partial COT_{error}}{\partial \dot{m}_{inlet}} \end{pmatrix}^{-1} * -f\begin{pmatrix} \beta \\ COT \\ \dot{w}_f \\ \dot{m}_{inlet} \end{pmatrix} \qquad (22)$$

This iterative process needs to be carried out until the error terms reduce to a low enough value.

3. CONCLUSION

This paper presented the methodology of carrying out on design and off design gas turbine performance simulation using enthalpy based formulations. The advantage of this formulation is that the variation of gas properties with temperature can be accounted for accurately. The change in concentration of different gaseous species after combustion can also be captured using the NASA CEA code that provides the equilibrium concentration of the different species of reactants and products. Two means of scaling maps found in literature were compared and the short comings of the second method were pointed out.

The current simulation program is currently being extended to account for twin spool configurations. Further, one dimensional models of compressor and combustor are being developed to increase the fidelity of the simulation.

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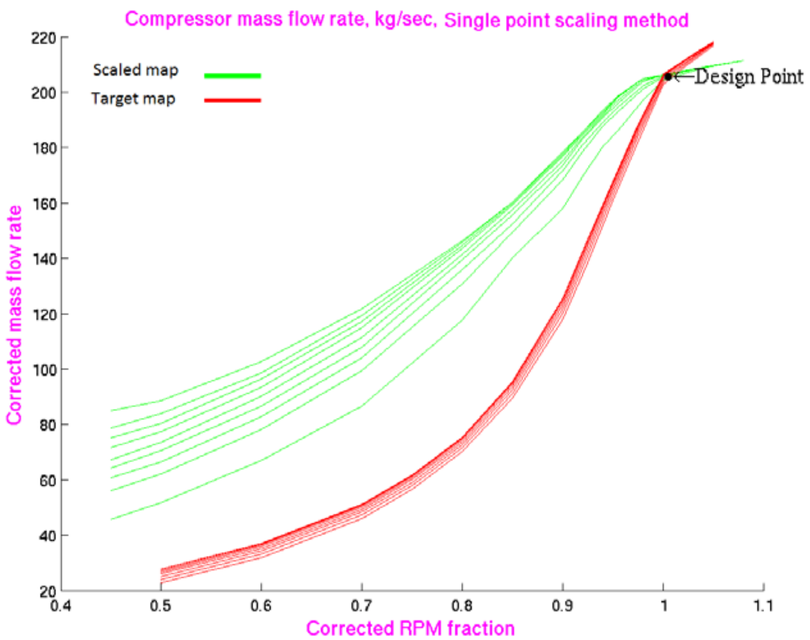

Figure 3: Comparison of scaled and target compressor mass flow rates, scaled using single point scaling method

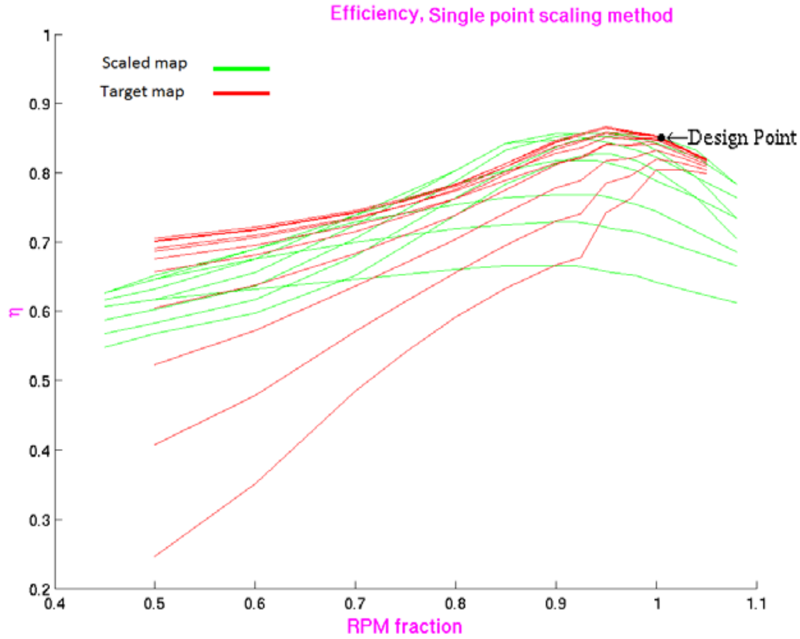

Figure 4: Comparison of scaled and target compressor efficiency, scaled using single point scaling method

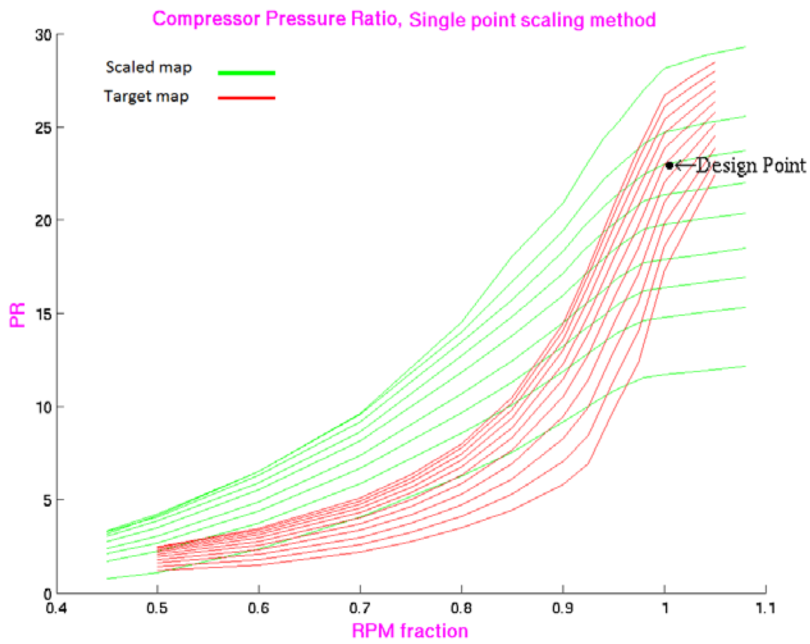

Figure 5: Comparison of scaled and target compressor pressure ratios, scaled using single point scaling method

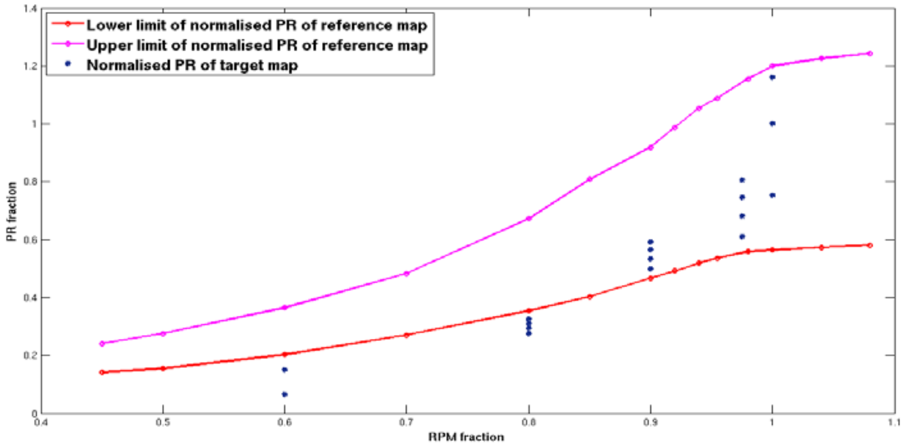

Figure 6: Normalised PR of the target engine, plotted over the upper and lower limits of the normalised PR of the reference map.

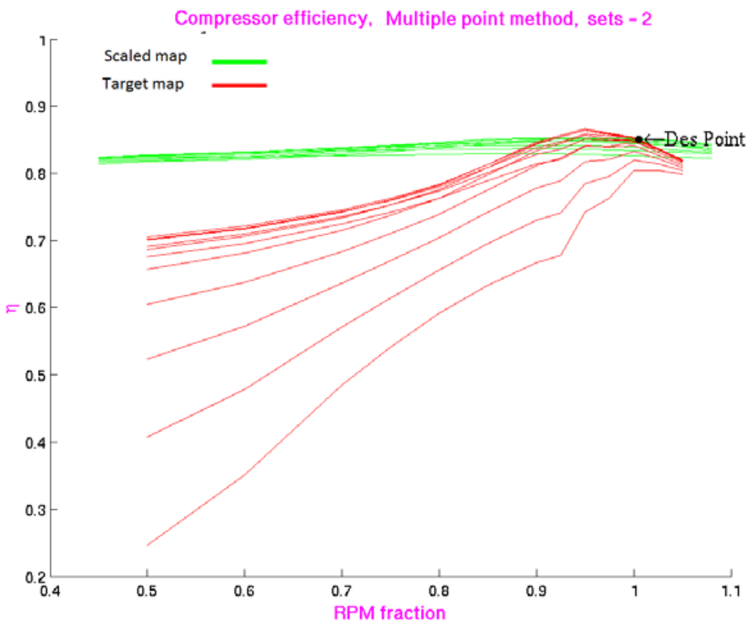

Figure 7: Comparison of scaled and target compressor efficiency maps, using two operating points in multipoint method.

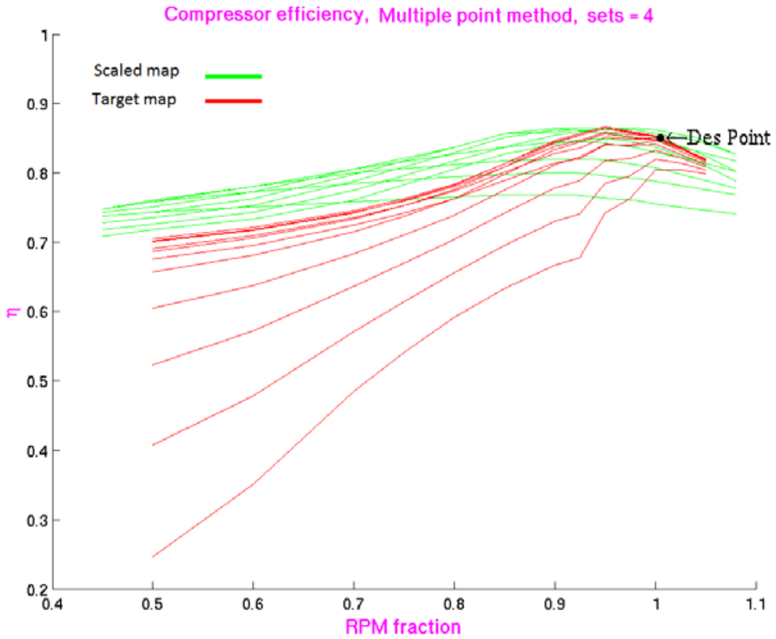

Figure 8: Comparison of scaled and target compressor efficiency maps, using 4 operating points in multipoint method.

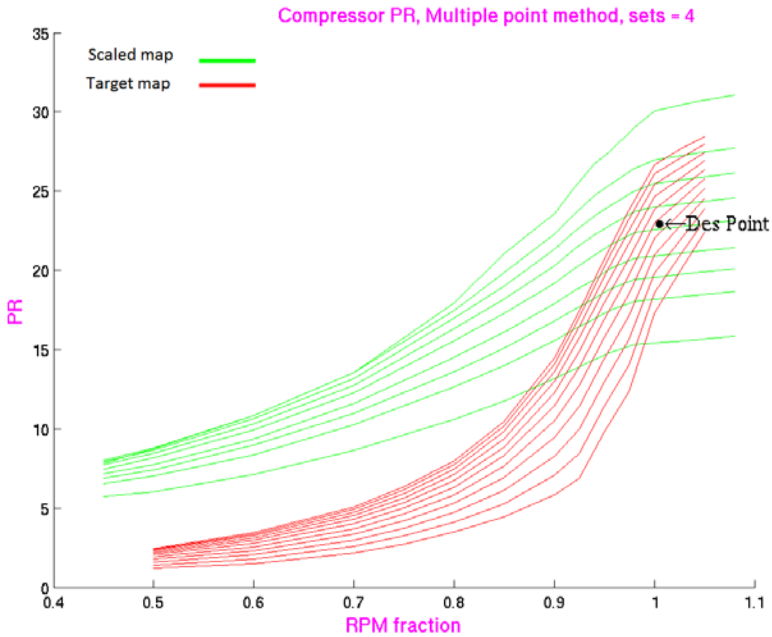

Figure 9: Comparison of scaled and target compressor pressure ratio maps, using two operating points in multipoint method.

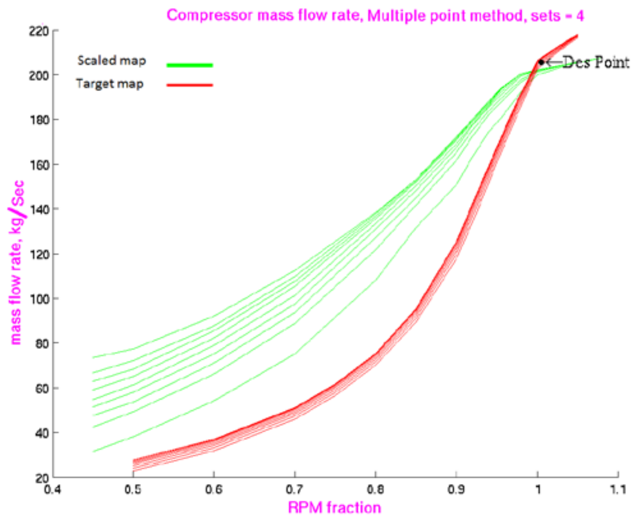

Figure 10: Comparison of scaled and target compressor mass flow rate maps, using 4 operating points in multipoint method.

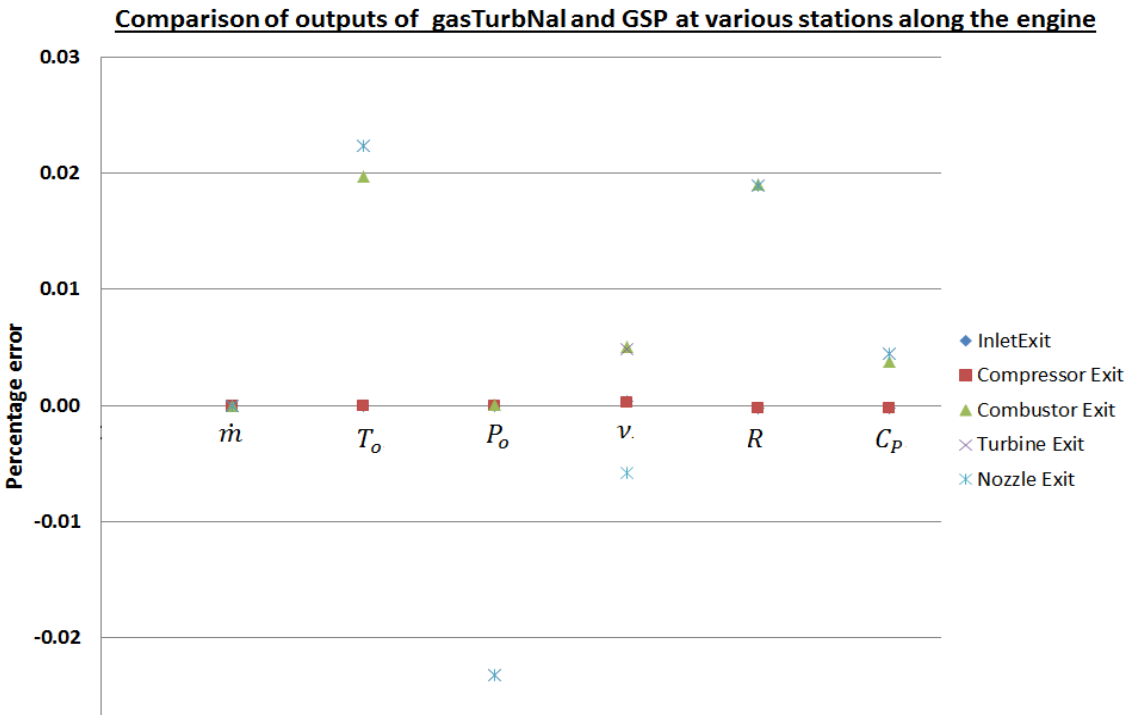

Figure 11: Comparison of gas path parameters given by GSP and gasTurbNal along the engine.