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Energy supply planning and supply chain optimization under uncertainty☆

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ABSTRACT

In energy supply planning and supply chain design, the coupling between long-term planning decisions like capital investment and short-term operation decisions like dispatching present a challenge, waiting to be tackled by systems and control engineers. The coupling is further complicated by uncertainties, which may arise from several sources including the market, politics, and technology. This paper addresses the coupling in the context of energy supply planning and supply chain design. We first discuss a simple two-stage stochastic program formulation that addresses optimization of an energy supply chain in the presence of uncertainties. The two-stage formulation can handle problems in which all design decisions are made up front and operating parameters act as 'recourse' decisions that can be varied from one time period to next based on realized values of uncertain parameters. The design of a biodiesel production network in the Southeastern region of the United States is used as an illustrative example. The discussion then moves on to a more complex multi-stage, multi-scale stochastic decision problem in which periodic investment/policy decisions are made on a time scale orders of magnitude slower than that of operating decisions. The problem of energy capacity planning is introduced as an example. In the particular problem we examine, annual acquisition of energy generation capacities of various types are coupled with hourly energy production and dispatch decisions. The increasing role of renewable sources like wind and solar necessitates the use of a fine-grained time scale for accurate assessment of their values. Use of storage intended to overcome the limitations of intermittent sources puts further demand on the modeling and optimization. Numerical challenges that arise from the multi-scale nature and uncertainties are reviewed and some possible modeling and numerical solution approaches are discussed.

1. Introduction

Energy has emerged as a major issue for the new millennium. With the accelerating energy consumption and consequent climate change, it has become a top priority to develop a sustainable long-term energy solution for the humanity. Nature and scope of the energy problem are extremely general and broad; a successful resolution is likely to require a carefully coordinated and tightly integrated system of multiple solution approaches, including the development of renewable energy sources, advancements in energy generation and processing technologies, smart supply chain management (*e.g.*, the demand-side control through variable pricing and smart grid technology), and well thought out policy-making. Systems engineers should serve at the forefront of meeting these challenges as the problem's complexity clearly calls for systems thinking and approaches.

In energy supply planning, coupling between investment/design decisions and operating decisions presents an interesting challenge. The first challenging aspect of it is the multi-scale nature, which arises as capacity investment and design decisions are typically made on a much coarser (and longer) time scale than operation decisions. Situations are not uncommon, in which investments into new energy capacities are reviewed on an yearly basis, whereas operations of existing capacities require decisions on a much faster time scale. The coupling between these decisions makes the overall decision problem a multi-scale one. The use of fine-grained time scales is needed in modern energy supply planning as there is a growing need to accurately assess true values of intermittent energy resources like wind turbines and solar panels. Separate from the need to handle multiple time scales is the need to capture and account for various types of uncertainties, which can be large and significant. There are the usual suspects like uncertain demands and feedstock supplies.

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In addition, because some of the relevant technologies are still in the process of maturing, many important parameters like processing costs and yields are highly uncertain. Finally, there are uncertainties regarding the future course of energy policies such as the imposition of a carbon tax.

This paper is intended to bring to attention various research issues that arise from the above-mentioned coupling in the context of energy supply planning and supply chain design problems. A particular focus is given to its multi-scale and stochastic nature. The paper will also review some of the promising problem formulations and solution approaches that exist in the current literature, highlighting their relative advantages and shortcomings. By doing this, it is hoped that the promising approaches are further investigated and alternative methods are developed, especially by our research community.

We start with a discussion of a simple two-stage stochastic program that addresses design and optimization of an energy supply chain. The two-stage formulation covers the case wherein design parameters are one-time 'here-and-now' decisions and operating parameters are 'recourse' variables that can be decided in response to realized values of the uncertain parameters for a particular operation period. The design of a biodiesel production network is used as an illustrative example.

The discussion then moves on to a more complex multi-stage, multi-scale stochastic decision problem in which periodic capacity investment decisions are made over a planning horizon spanning multiple decades, along with operating decisions made on a much finer time-scale, e.g., hour-to-hour. Energy policy modeling is introduced as an example. In the energy policy modeling problem, investments in power generation/storage capacities are to be reviewed on an yearly basis whereas coupled operation decisions like electricity production and dispatch are made on a much faster time scale, *e.g.*, hour-to-hour basis. For intermittent energy sources like wind and solar, coarse-graining is not desirable as averaging over a large time period tends to take out peaks and valleys, which present a major problem for dispatch. It has been shown that, for these types of energy resources, using time increments longer than an hour can lead to significant overassessment of their values [15]. It is commonly assumed that the market behavior is mimicked by cost minimization, so the production and dispatch decisions are modeled through some cost minimization in the overall planning model.

Since the energy policy model is to span a very large planning horizon, it can involve a very large optimization problem that current numerical solvers cannot handle. These and other numerical challenges that arise from the multi-scale, uncertain nature are reviewed and some promising modeling and solution approaches are discussed. Though stochastic programming can be a useful tool for two-stage problems, its use is severely limited in the context of multi-scale, multi-stage problems since the number of scenarios grows exponentially with respect to the number of stages. Approximate dynamic programming (ADP), which attempts to solve the Bellman's optimality equation with reasonable approximations to obtain a near-optimal decision policy, is suggested as a promising algorithmic strategy to handle this class of problems.

2. Literature review

The literature on the subject of energy supply planning and supply chain design is too vast to cover comprehensively. Hence no such effort is made here. Instead, we touch upon some small subset, which the author is familiar and considers relevant to the present discussion.

Given the substantial R&D investments that have gone into biofuel technologies during the past few decades, significant literature exist on the issue of optimal synthetic route selection and supply chain design for biomass and biofuel systems. Although lignocellulosic biomass has been touted as the most promising feedstock for next generation biofuel, significant uncertainties regarding their supply, price, demand, and technology greatly complicate the decision process for the potential investors of this technology. A proper decision process demands one to model the major uncertainties and analyze how they impact the design decisions and the expected profit. Design decisions determine the overall structure of the biofuel production network through capacity investment, production technology and location choices. They can be considered as one-time decisions, or multi-stage decisions to be made at various time points over a planning horizon. On top of these, there are short-term decisions on the procurement of biomass, processing, and distribution of products. These are recurring decisions that can be revised from one time period to another based on information at hand.

Decision models of increasing scope and sophistication have been devised, including those incorporating uncertainty. Geographical information systems (GIS) have been introduced to biomass supply chain studies in order to compute more accurately the expected supply of biomass in a given region and the transportation distances and related costs as well as to assess the impacts of spatial feedstock subtraction for different chain designs [19,20]. Dunnet et al. [5] presented a combined production and logistics model in order to optimize system configurations for a range of technological and supply/demand scenarios specific to European agricultural land and population densities. Eksioglu et al. [6] proposed a mathematical programming model to determine the number, sizes and locations of biorefineries and applied it to the State of Mississippi as a test case. Leduc et al. [16] presented a mixed integer linear programming (MILP) model to optimize the locations and sizes of methanol plants with heat recovery and gas stations in Austria.

Beyond these static models, planning model over multiple periods have been devised and tested. Huang et al. [9] proposed a mathematical model that minimizes the cost of an entire supply chain of biofuel from bio-waste feedstock fields to end users over an entire planning horizon, simultaneously satisfying demand, resource, and technology constraints. This model is tested on a case study to evaluate the economic potential and infrastructure requirements for bioethanol production from eight waste bio-mass resources in California. Zamboni et al. integrated the environmental aspects into the general framework of designing a bioethanol supply chain [27,28]. The design task is formulated as a mixed integer linear program (MILP) that accounts for the simultaneous minimization of the supply chain operating costs [27] as well as the environmental impact in terms of greenhouse gas (GHG) emissions [28]. Grossmann and Guillen-Gosalbez [7] reviewed major contributions in process synthesis and supply chain management including the handling of uncertainty and the multi-objective optimization of economic and environmental objectives and pointed to the need to develop sophisticated optimization and decision-support tools to help in exploring and analyzing diverse process alternatives under uncertainty. Mas et al. [18] devised a dynamic, spatially explicit MILP (mixed integer linear programming) modeling framework to optimize the design and planning of biomass-based fuel supply networks according to financial criteria in the face of market uncertainty. The model was tested on a real-world case study involving an emerging corn-based bioethanol production system in Northern Italy.

Significant efforts have been directed to the development of optimized energy planning models that allow the study of the economic effect of various investment options, policy changes, market changes, and technology developments. Many of these models include cost optimization as a part of the model, based on the assumption that cost minimization mimics the market behavior. Such models are often spatially aggregate models that stand between simple models analyzing the economics of different energy generation types in isolation and detailed network models used in operational planning for utilities and grid operators. Lawrence Livermore National Laboratory developed a deterministic energy planning model called META*Net [14], which is a spatially aggregate model capturing approximately 13 different energy sources covering different electrical power types, while satisfying 9 types of demands including different energy forms for industrial and residential sectors. Wallace and Fleten [26] presented a discussion of stochastic programming models to handle random information and decisions. Powell et al. [22] proposed a model of highest sophistication, called SMART (stochastic multi-scale model for the analysis of energy resources, technology, and policy), which is based on the concept of dynamic programming and can be applied to both deterministic and stochastic problems of a spatially aggregate or a disaggregate model. It uses a control theoretic framework to run simulations in hourly increments over multiple decades in order to derive near-optimal policies for annual capacity acquisitions and hourly dispatch decisions. The optimization model can be used for both long-term policy studies and economic analysis of a portfolio of energy generation and storage technologies.

3. Two-stage formulation of energy supply chain design and operation

3.1. Two-stage stochastic programming

Researchers have considered many different ways to consider uncertainty within the framework of mathematical programming. Stochastic programming (with recourse) is a general way to find optimal decisions prior to knowing the realization of some of the random variables so that the total expected costs of possible recourse actions (taken later) are minimized. Since the basic formulation was proposed by Dantzig [3], many researchers have developed extended formulations and solution procedures as a way to perform robust optimization [2,10,24,4].

The two-stage stochastic linear program takes the form of

$$\begin{aligned} \min_{x} \quad & c^T x + E_\omega\{Q(x,\omega)\} \\ & Ax = b \\ & x \geq 0 \end{aligned} \tag{1}$$

where

$$\begin{aligned} Q(x,\omega) \quad = \quad & \min_{y} d_\omega^T y \\ & R_\omega x + T_\omega y = g_\omega \\ & y \geq 0 \end{aligned} \tag{2}$$

$E_\omega\{\cdot\}$ is the expectation operator defined with respect to ω, which belongs to a set of scenarios Ω (with some discrete probability distribution) or a set of continuous random variables (with some probability distribution functions). Here x is the 'here-and-now' variable that must be decided in a non-anticipative manner, that is, independently of the realization of scenario, and y is the recourse variable that can be decided based on a realized outcome. Though a linear program is presented here for the sake of simplifying the discussion, it can be extended to include integer decision variables, which can be either 'here-and-now' variables or 'recourse' variables. In fact, in the case of supply chain design problems, many design decisions may be of 'yes' or 'no' nature and are naturally represented by a binary variable. In addition, some logical constraints may be formulated using integer decision variables.

In the case of a finite set of scenarios, we can write the above stochastic program as

$$E_\omega\left\{Q(x,\omega)\right\} = \sum_{\omega\in\Omega} p(\omega)Q(x,\omega) \tag{3}$$

where $p(\omega)$ is the probability of scenario ω. (If ω is a continuous random variable, evaluation of the expectation would involve an integral rather than a summation.) This means that the stochastic program can be reformulated as the following deterministic equivalent LP of a larger size:

$$\begin{aligned} \min_{x,y_\omega} \quad & c^T x + \sum_{\omega\in\Omega} p(\omega) d_\omega^T y_\omega \\ & Ax = b \\ & R_\omega x + T_\omega y_\omega = g_\omega \\ & x \geq 0,\ y_\omega \geq 0 \end{aligned} \tag{4}$$

Note that the above is a significantly larger LP since there is a separate y variable and also a separate set of constraint $Rx + Ty = g$, $y \geq 0$ for each $\omega \in \Omega$.

This formulation can be used to address a simple kind of supply chain design and optimization problems. In such a problem, x would correspond to design variables such as locations and size of procurement, processing and distribution centers, which lay out a basic structure of a network, and y to operation variables, such as the rate of material flows between the nodes, which can be adjusted from one time period to next in response to the realized values of demands and prices for that period. What this formulation is not able to address is a common case, in which the network structure needs to evolve with time. For example, it is unlikely that capital investments be made all at once; instead, capacities would be added in phases according to the evolving demand and price figures as well as various advances and breakthroughs in the related technologies.

3.2. Lignocellulosic-based biofuel supply chain example

3.2.1. Problem statement

To demonstrate the use of two-stage stochastic program in energy supply chain design and optimization under uncertainty, we discuss a supply chain network design problem for thermo-chemical conversion of biomass into biofuel within the Southeastern part of the United States, originally presented by Kim et al. [12] and later extended to include the effect of uncertainties [13]. The case study was developed with an industrial collaborator and the data used were representative estimates of various costs involved. Products and synthetic routes were fixed a priori. The processing was to be carried out in two steps: the first step ('conversion1') is Fast Pyrolysis to convert biomass into bio-oil, and the second step ('conversion2') is a Fischer Tropsch (FT) synthesis process that converts the bio-oil to a liquid fuel through a syngas intermediate. The rationale behind dividing the processing into two steps is that there may be significant savings in the transportation costs by pre-processing the bulky biomass into smaller volumes of bio-oil at processing locations close to the biomass supply sites before they are transported farther to the larger-scale FT plants.

The case study covers ten states (Oklahoma, Arkansas, Louisiana, Mississippi, Alabama, Tennessee, Georgia, Florida, South Carolina, and North Carolina). The particular processing option can utilize a wide array of biomass types as its feedstock, and five different biomass types (logging residuals, thinnings, prunings, grasses and chips/shavings) harvested at 30 biomass source locations are considered. There are 29 candidate locations for conversion1 plants, which convert biomass materials into three intermediate products (bio-oil, char, and fuel gas). Char and fuel gas are to be consumed locally as utility energy sources, and only bio-oil is transported to conversion 2 plants. There are 10 candidate sites for conversion 2 plants, where final products (gasoline and biodiesel) are to be produced and distributed to 10 final market sites for sale. Each plant can be sized from four different capacity options. The GIS was used to obtain distance matrices from the provided latitude and longitude information for each location, in order to calculate the transportation costs. Details of the problem as well as the key parameter values can be found in Refs. [12,13].

The supply chain network design and optimization problem can be summarized as follows:

- Thirty biomass sites are given where five biomass types (logging residuals, thinnings, prunings, grasses, and chips/shavings) are harvested to be used as a feedstock to conversion 1 plant.
- Twenty nine candidate sites are given for building conversion1 plants of four capacity options where three kinds of intermediate products (bio-oil, char and fuel gas) are manufactured to be used as feedstock or utility at conversion2 plants or as a utility locally.
- Ten candidate sites are given for building conversion2 plants of four capacity options where final products (gasoline and biodiesel) are manufactured and transported to the final markets or the intermediate products from the conversion1 plants can be used as a utility without producing the final products.
- 10 market locations, where the final products are sold, are given with certain maximum demands.

The objective is to determine (1) the optimal network structure (*i.e.*, to decide the number, location, and size of the two types of processing units) and (2) the optimal operation variables (*i.e.*, the amount of materials to be transported between the various nodes of the designed network) so that the overall profit is maximized while respecting the constraints associated with product demands. This optimization needs to account for significant uncertainties, which are to be expressed through a number of scenarios defined by different parameter values. The size of each scenario is substantial, which means that only a limited number of scenarios can be considered simultaneously in the mathematical optimization problem.

The following fourteen parameters were considered to have significant uncertainties or variability: Biomass availability for each biomass type; annualized capital cost of conversion1 and conversion 2 processing; acquisition cost for each biomass type; cost of transporting intermediate products; cost of transporting biomass; value of each intermediate product at conversion1 and conversion2 processing site; yield of final product from intermediate product at conversion2 processing; cost of transporting final products; yield of intermediate product from biomass at conversion1 processing; maximum demands; operating costs for conversion1 and conversion2 processing; sale price of each final product. The biomass availability for all biomass types were assumed to vary together and therefore its uncertainty was parameterized by a single varying parameter. The same was true for their acquisition costs and the sales prices of all products. Even if one considers the only the upper and lower extreme values of the 14 parameters plus the nominal parameters, one gets $2^{14} + 1 = 16,385$ parameter combinations, which are too many to handle in the two-stage stochastic program. Hence, it is necessary to identify those parameters that have major impact on the optimal profit. The sensitivity analysis was used on the nominal scenario based design to identify five dominant ones among the fourteen parameters. With the chosen 5 dominant parameters, $2^5 = 32$ scenarios, corresponding to the various combinations of extreme low and high values of each selected parameter, are constructed. A new optimal design (called scenario robust design) maximizing the overall profit is built by solving a two stage stochastic program that considers all the 32 plus the nominal scenario (33 in total) simultaneously.

The robustness of nominal design vs. scenario robust design was analyzed using Monte Carlo simulation over a broader set of scenarios, selected from the entire parameter space defined by the fourteen dimensional hypercube of the parameter ranges. Global Sensitivity Analysis (GSA) [23] is performed between the five selected parameters vs. the non-selected parameters to check whether the majority of the profit variability is captured by the variation in the five inputs rather than those that were excluded from the design scenarios.

3.2.2. Result

A MILP model was developed and tested for designing a single scenario based optimal network system. The MILP model was implemented on the commercial software GAMS^{TM} using the CPLEX MILP solver. First, an optimal supply chain network was designed based on the nominal single scenario with maximum demands. In the optimized design, biomass resources were transferred to 15 conversion1 processing locations (selected from the 27 biomass sites) and the bio-oil produced there was fed to 3 conversion2 processing units, which produced and delivered the final products to 10 final market locations.

Next the sensitivity of the network optimized based on the nominal scenario to varying parameters was analyzed. Change in the objective function value was calculated when each of the fourteen parameters was changed to the nominal plus or minus given percentage changes (−50%, −30%, −10%, 10%, 30%, and 50%). Changes of the sale price in the final market was seen to affect the overall profit the most followed by the conversion yield ratios of the both processing. Maximum demand and biomass availability had a strong effect on the overall profit when their values were at the lower ends of the ranges explored. The other 9 parameters were shown to have relatively smaller influence over the nominal design profit. The sensitivity analysis yielded five dominant parameters S = [yield of final product, yield of intermediate product, sale price of final product, maximum demand, biomass availability], which were to be considered in the subsequent robust design using the two-stage stochastic programming.

Scenarios were created by varying the five parameters by ±20%. It was assumed that availabilities of the five biomass types in 30 biomass sites varied simultaneously, not independently. Sale prices of the two final products for the 10 markets were also changed simultaneously, as was the maximum demand. In total, 33 scenarios (2^5 combinations of the extreme parameter values plus the nominal parameter values) were created. A two-stage stochastic program based on the previously formulated MILP was constructed to obtain another supply network design that considers the 33 scenarios simultaneously. In the new design, the first stage decisions on the size and location of the processing infrastructure are fixed, but the actual flows can be different in each scenario. It was assumed that the weight of each scenario was the same. In the optimized multiple scenarios network, biomass resources were transferred to 14 conversion1 processing locations selected from the 27 biomass sites. The bio-oil converted at conversion1 processing plants is fed to two conversion2 processing units. Gasoline and biodiesel are delivered to 10 final product demand locations from the two converion2 locations. *The multi-scenario-optimal processing network structure differed significantly from the single-scenario-optimal design* in that there were a number of new locations and several other locations seen in the previous design were eliminated.

In addition, the single-scenario optimal design was carried out for each of the 33 scenarios to maximize the overall profit. Hence, in this design, both the size and location of the processing infrastructure and the actual flows were allowed vary for each scenario. This is unrealistic in practice as the network structure must be decided before the realization of scenario but it was done to obtain a benchmark value, *i.e.*, an unattainable upper-bound of the profit value. Then, the robustness of the single nominal scenario design and multiple scenarios design were examined by comparing the profit values again the benchmark values. This way, deviations of the profit values from those of the optimal design can serve as indications of how robust the two designs are. *It was shown that the multiple scenarios design matched the optimal designs more closely in profit, implying improved robustness of the design obtained through the use of two-stage stochastic programming.* (see Kim et al. [13] for details)

4. Multi-stage, multi-scale formulation of energy supply chain design and operation

Though the two-stage stochastic programming formulation represents a viable approach for some energy supply chain design problems, it is limited by the ability to solve the resulting linear or mixed integer linear programs and therefore can address problems of only a modest number of scenarios. The two-stage formulation also assumes that network design decisions are made all at once in the beginning and therefore network structure remains fixed over time. The latter is a serious limitation as most practical energy supply chain design and optimization problems violate this assumption. For example, it is unlikely that capital investments into a biofuel supply chain will be put up all at once; instead, the network structure will evolve significantly over time as capacities, possibly using different technologies, are added in phases to address the growing demands, policy changes, and technological advancements.

In order to build a general formulation that can address the situation for a large majority of practical energy supply chain problems, we need a *truly multi-stage formulation*, wherein investment/design decisions are interspersed throughout the planning horizon. In addition, the period lengths for operation variables are generally much shorter than those for investment decisions, thus creating a multi-scale modeling and optimization problem. Take a power generation/distribution network as an example. With the increased presence of intermittent renewable energy sources like wind turbines, the electrical grid operation needs to be modeled on as fine-grained a time scale as hour to hour in order to represent their variabilities accurately, while structural decisions like addition of extra capacities are likely to follow a much coarser time scale, e.g., an year-to-year time scale. Since the planning horizon for energy capacities and policies may need to stretch over several decades to be meaningful, whereas a grid operation model may involve a cost minimization on an hourly scale, the sheer number of stages (*i.e.*, the number of hours in multiple decades) presents us with a daunting challenge.

4.1. Stochastic programming

In concept, stochastic programming with recourse generalizes to multi-stage problems straightforwardly. Beyond the two-stage problems, however, it is difficult to apply it because the number of branches often becomes too large for the computation to be tractable. This is especially true for the above-described energy supply chain design and optimization problems due to their multi-scale nature. This problem can be described pictorially in Fig. 1. Here the stage numbering is done using two indices, i and t. $i = 1, \ldots, P$ is the index for the longer time period (referred to as 'stage' hereafter) representing an interval between successive investment decisions. Within each stage i, there are N shorter time periods indexed by $t = , 1, \ldots, N$ representing an interval between successive operation decisions. $x_{i,0}$, $i = 1, \ldots, P$ denotes the decision variables pertaining to network structure (*e.g.*, capacity additions for various energy conversion technologies, capacity additions for various

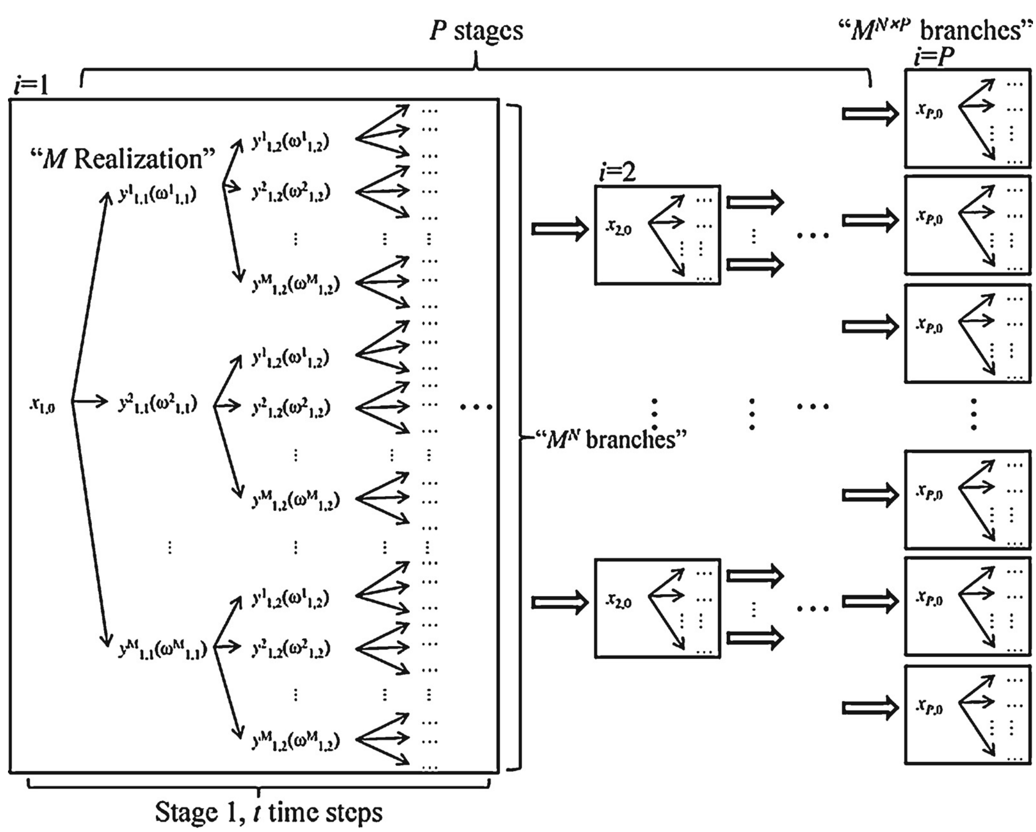

Fig. 1. Pictorial representation of stochastic program for multi-stage, multi-scale supply chain design and optimization problems.

storage types), which are decided before the ith stage commences and can be based on a realized scenario up to the decision point. Accordingly, $y_{i,t}$, $i=1, \ldots, P$, $t=1, \ldots, N$ represent decision variables pertaining to network operation (*e.g.*, dispatch decisions that govern the flow of energy from each source node to demand node in the network), which too can be decided based on realized scenario up to the decision point. $\omega_{i,t} \in \Omega$ represents the realization of the random variables for the tth time point of the ith stage. The flow balances for each node in the network as well as the material balances for the conversion or storage capacities are added as constraints.

From the diagram, it is obvious that the number of branches will grow exponentially, according to $O(M^{(N^*P)})$ where M is the number of possible outcomes for each realization. The problem will quickly become intractable as the number of stages, the number of time periods within each stage, and the number of scenarios grow. Note that N is typically a very large integer for multi-scale problems. This makes the use of stochastic programming all but impossible beyond two stage problems. One possible approach is to use aggregation to combine scenarios and periods so that the number of branches is dramatically reduced, but a systematic way to do this is lacking [2]. An alternative approach is to use dual information to generate cuts to approximate the impact of decisions made now on the future [25,8]

Another difficulty for stochastic programming is the need to express uncertainties in terms of a finite number of scenarios declared in advance. Some uncertainties may be more naturally represented by continuous random variables, or stochastic processes in the case that uncertain variables carry significant temporal correlations. In addition, there may be uncertainty types whose outcomes are affected by prior decisions.

4.2. Formulation as a Markov decision process

There are a number of control-theoretic approaches that are applicable to multi-stage stochastic decision problems like the one at hand. In order to use these techniques, it is convenient to have a state-oriented representation of the problem, such as a Markov Decision Process (MDP) representation. For our problem, a MDP can be formulated based on the following system description:

$$S_{i,t+1} = T_1(S_{i,t}, y_{i,t}, \omega_{i,t}), \quad t = 1, \ldots, N \tag{5}$$

and

$$S_{i+1,1} = T_2(S_{i+1,0}, x_{i+1,0}), \quad i = 0, \ldots, P-1 \tag{6}$$

with the transition from stage i to stage $i+1$ given by

$$S_{i+1,0} = S_{i,N+1} \tag{7}$$

$S_{i,t}$ is the state vector that contains all the relevant information available at the beginning of time period (i, t), including the available capacity for each energy conversion and storage type, the total amount of energy stored in each storage node, and the information revealed about uncertain variables (*i.e.*, realized values of the random variables) such as the demand, climate, the state of technology, etc. $\omega_{i,t}$ is an independent random variable sequence, which may follow either continuous or discrete probability distributions. T_1 and T_2 are transition functions defined appropriately. We note that, although they are represented conveniently by symbols T_1 and T_2 in our exposition, in reality, they can be very complex functions, especially when dimensions of the state and the decision variables are high. Fortunately, it is not always necessary to declare them mathematically; instead, just an algorithm to compute these transitions may be needed.

If we represent the cost functions associated with incremental capacity acquisition and energy dispatch as $Q_{i,0}(S_{i,0}, x_{i,0})$ and $R_{i,t}(S_{i,t}, y_{i,t})$, then our objective can be expressed as

$$\min_{\pi\in\Pi} E\left\{\sum_{i=1}^{P}\left(Q_{i,0}(S_{i,0}, X^{\pi}_{i,0}(S_{i,0})) + \sum_{t=1}^{N} R_{i,t}(S_{i,t}, Y^{\pi}_{i,t}(S_{i,t}))\right)\right\} \tag{8}$$

where $X^{\pi}_{i,0}$ is the decision policy mapping state $S_{i,0}$ to decision $x_{i,0}$ and $Y^{\pi}_{i,t}$ that mapping $S_{i,t}$ to $y_{i,t}$. Π represent all admissible policies, which in general can be arbitrarily complex functions of the state. The objective, of course, is to find the optimal policy among all the admissible ones.

4.3. Dynamic programming

There are a number of approaches to solving the multi-stage stochastic optimal control problem of type (8). They include myopic policies, rolling horizon policies, and stochastic programming. The first two yield substantially suboptimal results and the stochastic programming has the aforementioned limitations. Another well-known optimal control approach is dynamic programming, which can yield a stochastic optimal policy in principle and but presents us with a computational hurdle called 'curse of dimensionality.' Define the *value function* $J_{i,t}(S_{i,t})$ to represent the optimal *cost-to-go* (the sum of the stage-wise costs from time period (i, t) until the end of the horizon, with the system starting from the state $S_{i,t}$). The value function satisfies the so called *Bellman equation*, which can be expressed for our problem as

$$J_{i,0}(S_{i,0}) = \min_{x_{i,0}}(Q_{i,t}(S_{i,0}, x_{i,0}) + E\{J_{i,1}(S_{i,1})\}) \tag{9}$$

and

$$J_{i,t}(S_{i,t}) = \min_{y_{i,t}}(R_{i,t}(S_{i,t}, y_{i,t}) + E\{J_{i,t+1}(S_{i,t+1})\}) \tag{10}$$

where $i=1, \ldots, P$, $t=1, \ldots, N$, and $J_{i,N+1} = J_{i+1,0}$. Finding an optimal decision policy over the horizon (*i.e.*, $[X^{\pi}_{i,0}, (Y^{\pi}_{i,t})_{t=1,\ldots,N}]_{i=1,\ldots,P}$) amounts to obtaining $J_{i,t}$ satisfying the above but the fact that $J_{i,t}$ can be an arbitrarily complex function of $S_{i,t}$ presents us with computational and storage hurdles. In principle, the value of $J_{i,t}$ must be found for each $S_{i,t}$, the cardinality, of which the underlying space can be huge. In addition, equations for successive entries in the value table are coupled since $J_{i,t+1}(S_{i,t+1})$ appear in the RHS of the equation for $J_{i,t}$, therefore involving the state transition function. It can take many iterations (multiple sweeps across the entire state space and over the entire horizon) before they converge to optimal solutions.

Much work in this area has focused on reducing the computational load of dynamic programming down to a reasonable level. In the last two decades or so, the community of 'approximate dynamic programming (ADP)' has been formed to address this challenge through the use of simulation and function approximation. The following developments are particularly relevant for the multi-scale energy supply chain optimization problem discussed here:

- Significant computational savings can be achieved by limiting the value function to a particular functional form. [21] advocates the use of the following *separable, piecewise linear approximation*, which he found to work well for a large class of resource allocation problems including energy policy modeling:

$$\tilde{J}_{i,t}(S_{i,t}) = \sum_{j=1}^{\dim(S)} \tilde{J}^{j}_{i,t}(S^{j}_{i,t}) \tag{11}$$

where $\tilde{J}^j$ is the contribution by the jth element of the state vector to the approximate value function $\tilde{J}$. Each $\tilde{J}^j$ is assumed to be piecewise-linear and convex. The superscript $\tilde{}$ is used to emphasize the approximate nature of the resulting value function due to the restriction of its functional form.

- [21] also has advocated the use of *post-decision state based value function*. Let us define $S_{i,t}^{\text{post}}$ is the post-decision state, which is a fictitious state defined by

$$S_{i,t}^{\text{post}} = E\{S_{i,t+1}|S_{i,t}, y_{i,t}\} \tag{12}$$

Post-decision state is a state resulting from the deterministic transition only; hence, it is limited to cases where the state transition can be divided into successive deterministic and stochastic steps. Most interesting problems satisfy this requirement. The physical state $S_{i,t+1}$ can be obtained by adding the effect of the realized random variable $\omega_{i,t}$ to $S_{i,t}^{\text{post}}$.

Now let us define the post-decision-state-based value function as

$$\tilde{J}_{i,t}^{\text{post}}(S_{i,t-1}^{\text{post}}) = E\{\tilde{J}_{i,t}(S_{i,t})|S_{i,t-1}\} \tag{13}$$

Note that indexing has changed such that $\tilde{J}_{i,t}^{\text{post}}$ is now a function of $S_{i,t-1}^{\text{post}}$, which is $S_{i,t}$ without the stochastic component of the transition being added. With this, the Bellman equation changes to

$$\tilde{J}_{i,0}^{\text{post}} = E\{\min_{x_{i,0}}(Q_{i,t}(S_{i,0}, x_{i,0}) + \tilde{J}_{i,1}^{\text{post}})|S_{i-1,N}^{\text{post}}\} \tag{14}$$

and

$$\tilde{J}_{i,t}^{\text{post}} = E\{\min_{y_{i,t}}(R_{i,t}(S_{i,t}, y_{i,t}) + \tilde{J}_{t+1}^{\text{post}})|S_{i,t-1}^{\text{post}}\} \tag{15}$$

Note that the reformulation has now placed the expectation operator outside the minimization. This enables the use of a deterministic optimization solver to solve the minimization.

- The expectation operator can be replaced by many iterations of stochastic simulations and update of the value function $\tilde{J}_{i,t}^{\text{post}}$ over the horizon. For this, initialize the value function, $[\tilde{J}_{i,t}^{\text{post}}]_{t=1,\ldots,N}$, with some appropriate *initial approximations*. (One can set them to zero if no other information is available.) Then, in each iteration, one can start with some sampled initial state and simulate forward in time, solving the following decision problem at each time step $(i, t)_{i=1,\ldots,P,t=1,\ldots,N}$ using the approximate value function of the future time step available (first, the initial approximations and then the approximations from the previous iteration):

$$\min_{x_{i,0}}(Q_{i,0}(S_{i,0}, x_{i,0}) + \tilde{J}_{i,1}^{\text{post}}) \tag{16}$$

$$\min_{y_{i,t}}(R_{i,t}(S_{i,t}, y_{i,t}) + \tilde{J}_{i,t+1}^{\text{post}}) \tag{17}$$

Note that, as before, $\tilde{J}_{i,N+1}^{\text{post}} = \tilde{J}_{i+1,0}^{\text{post}}$, which couples Eq. 17 to the solution of Eq. (16) of the next stage (but from the previous iteration). This strategy is repeated for a large number of iterations, with the *update of the value function* occurring after each iteration. Since $\tilde{J}_{i,t}^{\text{post}}$ is limited to separable, piecewise-linear, and convex functions, the optimization solved at each time step is a relatively small (MI)LP, which can be handled efficiently by a standard commercial solver like CPLEX$^{\text{TM}}$. The duality of LP can be used to further simplify the update of the value function by using the concept of 'marginal values'. The iterative updating allows us to form an approximation of the expectation operator, which is now sitting outside thanks to the use of the post-decision state formulation.

4.4. Case study: electricity generation and distribution network design

Powell et al. [22] presents an electrical energy supply chain network design problem depicted in Fig. 2. It is a spatially aggregate case in which annual capacity acquisitions for four different energy generation options (*i.e.*, coal, wind, solar, and nuclear) are to be planned and the generated electrical energy are to be dispatched on an hourly basis according to demands of the various sectors (residential, commercial, and export). Storage through a hydro reservoir is available with a certain upper limit. The use of hourly time increment is needed in order to avoid the adverse effect of averaging out the peaks and valleys, which leads to an overassessment of the value of renewable sources like wind turbines. The planning model is to span a 30-year horizon, which translates into 262,800 hourly time periods ($i = 1, \ldots, 30$, $t = 1, \ldots, 8760$). They used separable, piecewise-linear approximation of the value functions with respect to the various resource states (*i.e.*, the current capacity for each generation option) as well as the amount of energy stored in the hydro reservoir. They also adopted the post-decision-state formulation of the dynamic programming and used repeated forward Monte-Carlo simulations (while solving the linear programs at each hourly increment) to iteratively update the value function sequence over the 30-year horizon. They called the overall scheme a Stochastic Multi-scale model for the analysis of energy resources, technology and policy (SMART).

They tested the three different policies against various stochastic realizations and compared the average performance. The three polices were: 'MYOPIC' using a single step optimization, which is equivalent to setting all the value functions to zero; 'OPT', which represents the optimal solution to the deterministic problem obtained by solving a large linear program cast over a large planning horizon; and 'ADP', which is the solution produced by using the approximate dynamic programming approach of SMART. Note that 'OPT' is an idealistic (but not practical) solution as it requires precognition, *i.e.*, realization of the stochastic variables over the entire planning horizon is known ahead of decisions.

First, using a deterministic version of the problem, they performed a series of comparisons of the ADP objective function and solution against those of linear programming. They found that solution of the linear program with a horizon of four years (35,000 time periods) took more than 20 hours using CPLEX Version 12 on a 2.5 GHz Intel Xeon process with 64GB of RAM, with the solution time growing superlinearly with the horizon size. They also found that ADP gave solutions within much less than 1% loss in performance using 300 iterations (*i.e.*, simulation runs) with computation time less than an hour. When compared to the MYOPIC policy, the ADP solutions gave several hundred fold improvement in performance in most cases. Another evidence that ADP achieved a near optimum was that for various demand patterns (*e.g.*, constant, sinusoidal, random) tried, hydro storage profiles obtained from the ADP solution matched very closely the optimal profiles produced by solving the LP. One major attractive feature of ADP is that its computational load increases linearly with the number of time periods and the number of iterations. Hence, to solve the full problem with the horizon size of 30 years, the computation time required would be under 8 hours, which is reasonable. On the other hand, even with the capabilities of modern optimization software, it would be nearly impossible to solve the linear program of an hourly increment covering the 30 year time span.

Next, they considered the case where the precipitation (*e.g.*, rainfall) was stochastic. They generated 50 different precipitation scenarios and simulated the ADP model under these scenarios to train the value function approximations. They showed that the ADP solution produced a hydro storage profile that had the intuitive behavior of building up storage earlier in the season to prepare for

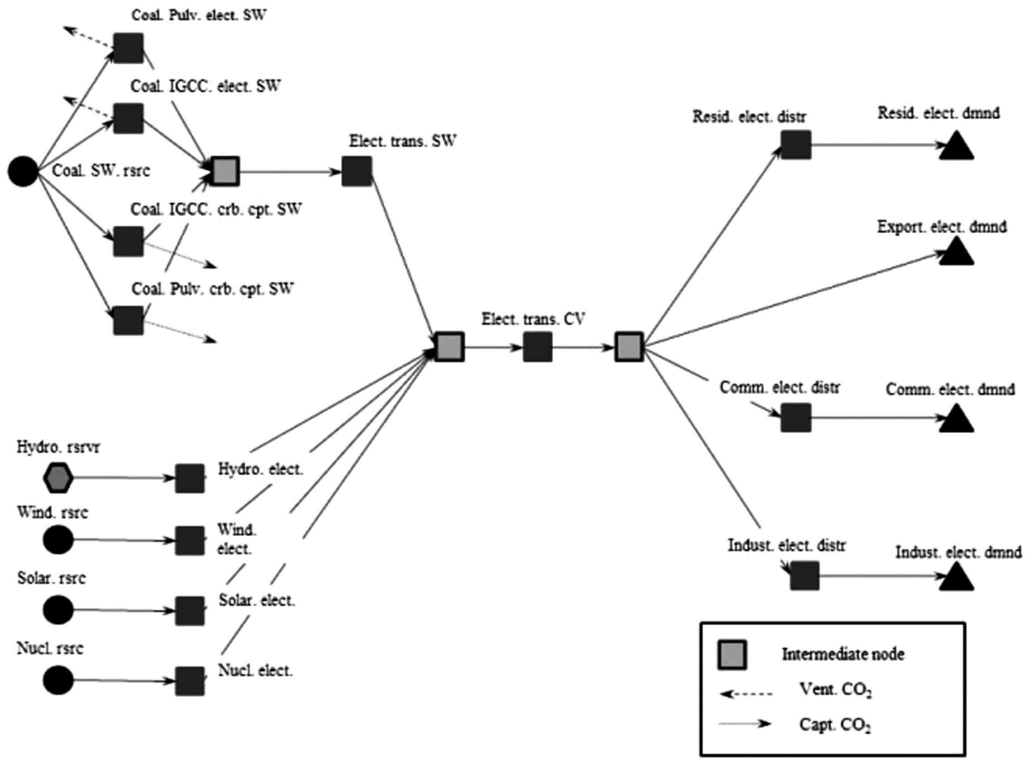

Fig. 2. Electrical energy supply chain network studied by Powell et al. [22].

potentially lower rainfalls later in the season. This is in contrast with the optimal solution obtained with the deterministic rainfall profile (assumed to be known in advance), which held the reservoir levels at lower levels until later in the season. Such a solution would not be robust in the stochastic case. This demonstrates the weakness of models that assume advance knowledge of uncertain information over the entire horizon.

5. Conclusion

Energy supply chain optimization presents a major challenge for system engineers due to the multi-scale nature and significant uncertainties. For problems in which an entire network structure is to be fixed a priori and uncertainties can be described by a relatively small number of scenarios, the two-stage stochastic programming framework can provide solutions. Potential benefits from considering multiple scenarios in the design were demonstrated by presenting a case study that involves a biofuel supply chain network design problem. For most practical problems where capital investments are made incrementally over a time horizon spanning multiple decades, however, a full multi-stage formulation is needed. Energy supply chain optimization problems are particularly challenging as the time scale for varying operating variables may need to be much more fine-grained that that for making capital investment decisions. The stochastic programming approach can be generalized to multi-stage problems in concept but a computational bottleneck is reached quickly as the number of scenarios grows exponentially with the number of stages. Stochastic optimal control techniques like dynamic programming may hold potentials but they require a state-transition-based formulation of the problem (*e.g.*, Markov Decision Process). This is not necessarily a disadvantage as many common features of uncertainties like autocorrelations and decision-dependent outcomes can be modeled naturally by state transitions. State-transition-based formulations are also conducive to simulation-based optimization techniques as certain state transitions, which may be too complex to be declared mathematically (as required by the mathematical programming approaches), may be easily computable within a computer simulation environment.

Dynamic programming demands solving the Bellman's optimality equation to find the optimal value functions. This presents a computational hurdle known as 'curse of dimensionality.' Approximate dynamic programming is presented as a computationally viable approach to overcome this hurdle. An annual capacity acquisition problem combined an hourly production / dispatch problem for an aggregate model of a moderate-sized electrical power generation / distribution network was used to demonstrate that (1)ADP can achieve a near-optimal solution for large deterministic planning problems with hundreds of thousands of time periods, and (2)it can yield robust solutions to stochastic problems of similar sizes. The fact that ADP is about the only available technique that can handle multi-stage stochastic decision problems of such size makes further research and investigation of it justifiable.

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