IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received March 24, 2019, accepted April 9, 2019, date of publication April 16, 2019, date of current version April 29, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2911632

Modeling of Unit Commitment With AC Power Flow Constraints Through Semi-Continuous Variables

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This work was supported in part by the National Basic Research Program of China (973 Program) under Grant 2013CB228205, and in part by the Project of Guangdong Electric Power Trading Center Co. under Grant GDKJXM20172986.

ABSTRACT Unit commitment is fundamental to power system operations, and the problem model is more precise with the consideration of AC power flow constraints. However, unit commitment with AC power flow constraints (ACUC) is intractable, and new formulations and methods that can improve the solution efficiency are still desired. In this paper, we propose a new optimization formulation for ACUC that is based on the inherent semi-continuous variables. Instead of using auxiliary binary variables that indicate the on/off states of generating units, we directly utilize the semi-continuous variables to describe the operation levels of units, and the overall problem is reformulated without changing the original logical relation except for few and close approximations. Therefore, it reduces the number of variables and simplifies the search space compared with the commonly used binary paradigm. As an important result, the proposed formulation can leverage commercial solvers to obtain a solution more effectively. The numerical experiments on the IEEE 39- and 118-bus test systems and a large-scale 739-bus system demonstrate the effectiveness of our model and method.

INDEX TERMS AC power flow, power system modeling, semi-continuous variable, unit commitment.

I. INTRODUCTION

Unit commitment (UC) [1]–[3] is the determination of a cost-effective generation schedule of units that is subject to satisfying load demand and other constraints such as physical limits of generators. In recent decades, more attention has been paid to security-constrained unit commitment (SCUC) or network-constrained unit commitment (NCUC), which takes the network constraints of the power system, such as power flow and bus voltage, into account. A UC model considering the network constraints was constructed in [4], which also included the demand-side resources and carbon emissions trading. A new SCUC formulation with binary variables modeling the active or inactive transmission line status was developed in [5]. A two-stage formulation for the day-ahead network-constrained unit commitment with demand response was proposed in [6], whose objective was to maximize the social welfare. The continuous-time unit commitment problem and its applications were studied in [7]–[9], which captured sub-hourly variations and ramping of load instead of the piecewise constant load curve. A UC problem with dynamic ramp limits, which were a function of the unit's generating output, allowing intra-period ramp-rate changes was considered in [10] so that the units' flexibility could be managed more efficiently. Typically, it leads to two categories of SCUC models with respect to different representations of the network: UC with DC power flow constraints (DCUC) [11], [12] and UC with AC power flow constraints (ACUC) [6], [13]. DCUC is easier to handle by using linear network constraints, but it ignores the effects of reactive power and bus voltages, and thus it requires an ex-post correction considering AC power flow constraints [14]. As a consequence, further rescheduling processes may be undertaken [15]. ACUC is a more precise model reflecting the system conditions, and it is meaningful as the operating conditions become more stressed with the development of various interconnected power systems, which, for example, may cause voltage problems [16]. However, ACUC is generally hard to solve. As reliability is essential to system operation and a method of processing ACUC is still needed, we focus on ACUC in this work.

Formulations of UC are usually a class of mixed integer programming (MIP) problems, where the discrete variables

The associate editor coordinating the review of this manuscript and approving it for publication was Yunfeng Wen.

are basically the binary variables representing the on/off state of each generator at each time interval [17]. Several studies have introduced other related binary variables to identify startup and shutdown changes [18], [19]. Instead of directly representing the on/off status, Atakan *et al.* [19] defined a new set of binary variables to indicate whether a generator remains operational in a time slot, and proposed a new state transition UC formulation. UC formulations with multi-set binary variables are often intended to improve the tightness (distance between relaxed and integer solutions) [18] compared to formulations using single-set binary variables. However, these problems can be computationally intensive with more binary variables [17].

To reduce the combinatorial complexity of UC, Tumuluru *et al.* [20] did not treat each on/off state as a binary variable, but defined two feasible sub-paths for each generator at equal intervals, which were the sequence of on/off statuses satisfying pre-calculated minimum up and down time constraints. Each sub-path was associated with one binary variable to show whether it was selected. This method required several pre-calculations and operations to link the two sub-paths. Clustered UC was developed by clustering similar and/or identical units [21], [22] to reduce the computational cost. An integer variable with $N + 1$ states was assigned to each cluster of N units (0 for offline). The combinatorial state space was reduced, but this method was not that beneficial when solving problems with a detailed network representation, as clustering might be limited [22]. Convex relaxation through semi-definite programming (SDP) relaxed the binary condition of an on/off state variable and expressed it as a quadratic equation [23], [24]. The SDP model might offer a solution against the integrality condition when the relaxations were not exact, thus other procedures were needed to obtain the final integer solution [23], [24].

All of the above papers that reformulated UC introduced auxiliary discrete variables to describe the states of units in addition to primary power-generation variables. In this paper, we present a new ACUC formulation directly based on its inherent semi-continuous variable [25], i.e., the power output of a generator which takes any value of $0 \cup [P_{i\min}, P_{i\max}]$ that has a minimum output $P_{i\min}$ if it is scheduled online ($P_{i\max}$ is the maximum output). Unlike existing models, which usually give special attention to the on/off states of generators, no auxiliary binary variables are used here, hence the reduction of the number of decision variables and the simplification of the search space. It can leverage commercial solvers to provide a solution more effectively than the widely used ACUC model.

The rest of the paper is organized as follows. In Section II, we present a common ACUC formulation based on binary variables in the literature. In Section III, we illustrate the relation between a binary variable and semi-continuous variable in ACUC, and then discuss the new formulation. Case studies on two IEEE test systems are presented in Section IV, and Section V presents our conclusions.

II. BINARY-VARIABLE-BASED ACUC FORMULATION

The binary-variable-based ACUC formulation we present here is mainly based on [23], [24], and [26], with small refinements such that the binary variables simply appear in linear terms.

In the following, i and Ω_G are respectively the index and set of generators; t and τ are indices of time intervals; Ω_T is the set of time intervals; N_T is the total number of time intervals; n and m are indices of buses; and Ω_N is the set of buses. $P_{i,t}$ and $Q_{i,t}$ are variables of the active and reactive power outputs, respectively, of unit i at time interval t; $e_{n,t}$ and $f_{n,t}$ are variables of the real and imaginary parts, respectively, of the voltage of bus n at time interval t; $u_{i,t}$ is a binary variable representing the commitment state of unit i at time interval t, which equals 1 if online and 0 otherwise; and $C^{su}_{i,t}$ is a variable that denotes the startup cost of unit i at time interval t. $P_{i\min}/Q_{i\min}$ and $P_{i\max}/Q_{i\max}$ are the lower and upper limits, respectively, of active/reactive power outputs of unit i; R^U_i and R^D_i are the maximum ramp-up and ramp-down rates, respectively, of unit i; $\overline{T}_i$ and $\underline{T}_i$ are the minimum online and offline time, respectively, of unit i; $D^P_{n,t}$ and $D^Q_{n,t}$ are the active and reactive loads, respectively, at bus n at time interval t; $V_{n,\min}$ and $V_{n,\max}$ are the lower and upper limits, respectively, of the voltage at bus n; $I_{nm,\max}$ is the maximum transmission current through line nm; Ω_L is the set of lines; SR_t is the system spinning reserve requirement at time interval t; and $G_{n,m}$ and $B_{n,m}$ are the real and imaginary parts, respectively, of the nmth element of the admittance matrix.

III. OBJECTIVE

The objective is to minimize the total production and startup costs of all generating units during the given time horizon:

$$\min \sum_{t\in\Omega_T}\sum_{i\in\Omega_G}(a_i u_{i,t} + b_i P_{i,t} + c_i P_{i,t}^2 + C^{su}_{i,t}), \qquad (1)$$

where a_i, b_i, and c_i are production cost coefficients. The startup cost is incurred if unit i is brought online at time interval t [26]:

$$C^{su}_{i,t} \geq \sigma_i(u_{i,t} - u_{i,t-1}) \quad i \in \Omega_G,\ t \in \Omega_T \qquad (2)$$

$$C^{su}_{i,t} \geq 0 \quad i \in \Omega_G,\ t \in \Omega_T, \qquad (3)$$

where σ_i is the startup cost coefficient of unit i.

A. CONSTRAINTS

1) AC POWER FLOW EQUATIONS

$$P_{n,t} - D^P_{n,t} - \sum_{m\in\Omega_N}[e_{n,t}(e_{m,t}G_{n,m} - f_{m,t}B_{n,m}) + f_{n,t}(f_{m,t}G_{n,m} + e_{m,t}B_{n,m})] = 0 \quad n \in \Omega_N,\ t \in \Omega_T \qquad (4)$$

$$Q_{n,t} - D^Q_{n,t} - \sum_{m\in\Omega_N}[f_{n,t}(e_{m,t}G_{n,m} - f_{m,t}B_{n,m}) - e_{n,t}(f_{m,t}G_{n,m} + e_{m,t}B_{n,m})] = 0 \quad n \in \Omega_N,\ t \in \Omega_T. \qquad (5)$$

2) LOWER AND UPPER LIMITS OF GENERATING UNITS

$$P_{i\min}u_{i,t} \le P_{i,t} \le P_{i\max}u_{i,t} \quad i \in \Omega_G,\ t \in \Omega_T \tag{6}$$

$$Q_{i\min}u_{i,t} \le Q_{i,t} \le Q_{i\max}u_{i,t} \quad i \in \Omega_G,\ t \in \Omega_T. \tag{7}$$

3) RAMP RATE CONSTRAINTS

$$P_{i,t} - P_{i,t-1} \le R_i^U u_{i,t-1} + P_{i\min}\left(1 - u_{i,t-1}\right) \quad i \in \Omega_G,\ t \in \Omega_T \tag{8}$$

$$P_{i,t-1} - P_{i,t} \le R_i^D u_{i,t} + P_{i\min}\left(1 - u_{i,t}\right) \quad i \in \Omega_G,\ t \in \Omega_T. \tag{9}$$

Specifically, $P_{i\min}$ corresponds here to the startup and shutdown ramp rate [23] which is smaller than R_i^U and R_i^D.

4) MINIMUM ONLINE AND OFFLINE DURATION TIME CONSTRAINTS

Once a unit is scheduled online/offline, it must remain in that state for a specific period of time [24]:

$$u_{i,t} = 1, \quad t \in [1, \max(0, \overline{T}_i - \overline{T}_i^0)],\ i \in \Omega_G \tag{10a}$$

$$u_{i,t} - u_{i,t-1} \le u_{i,\tau},\ \tau \in [t+1, \min(t + \overline{T}_i - 1, N_T)],\ t \in [\max(0, \overline{T}_i - \overline{T}_i^0) + 1, N_T], \quad i \in \Omega_G \tag{10b}$$

$$u_{i,t} = 0, \quad t \in [1, \max(0, \underline{T}_i - \underline{T}_i^0)],\ i \in \Omega_G \tag{11a}$$

$$u_{i,t-1} - u_{i,t} \le 1 - u_{i,\tau},\ \tau \in [t+1, \min(t + \underline{T}_i - 1, N_T)],\ t \in [\max(0, \underline{T}_i - \underline{T}_i^0) + 1, N_T], \quad i \in \Omega_G, \tag{11b}$$

where $\overline{T}_i^0$ and $\underline{T}_i^0$ denote the initial up and down duration time, respectively. Note that if a unit has been kept online (offline) before the current scheduling horizon, then $\overline{T}_i^0(\underline{T}_i^0)$ is the initial up (down) time and $\underline{T}_i^0(\overline{T}_i^0)$ is recorded as $\underline{T}_i(\overline{T}_i)$. For example, $\overline{T}_i^0 = 3$ and $\underline{T}_i^0 = T_i$ indicates the unit has been kept online for 3 hours before the scheduling.

5) BRANCH CURRENT LIMITS

$$(G_{n,m}^2 + B_{n,m}^2)[e_{n,t}^2 + f_{n,t}^2 + e_{m,t}^2 + f_{m,t}^2 - 2(e_{n,t}e_{m,t} + f_{n,t}f_{m,t})] \le I_{nm,\max}^2 \quad nm \in \Omega_L,\ t \in \Omega_T. \tag{12}$$

6) BUS VOLTAGE LIMITS

$$V_{n,\min}^2 \le e_{n,t}^2 + f_{n,t}^2 \le V_{n,\max}^2 \quad n \in \Omega_N,\ t \in \Omega_T. \tag{13}$$

The imaginary part of the voltage at the slack bus is set to zero.

7) SPINNING RESERVE CONSTRAINTS

For any time period, the spinning reserve provided by all units should meet the system spinning reserve requirement [23]:

$$\sum_{i \in \Omega_G} (P_{i\max}u_{i,t} - P_{i,t}) \ge SR_t \quad t \in \Omega_T. \tag{14}$$

IV. PROPOSED FORMULATION

A. BINARY AND SEMI-CONTINUOUS VARIABLES

Before discussing the proposed formulation, we will briefly describe the relation between binary and semi-continuous variables in ACUC. From (6), if a unit is scheduled online, i.e., $u_{i,t}$ equals 1, it outputs active power within $[P_{i\min}, P_{i\max}]$; otherwise, $u_{i,t}$ equals 0 and the active power output is 0. In this way, the binary variable $u_{i,t}$ indicates whether a unit is online. But $u_{i,t}$ is actually introduced as an auxiliary variable for ACUC. We can certainly tell the on/off state of a unit by $P_{i,t}$ alone: it is online when $P_{i,t} \in [P_{i\min}, P_{i\max}]$, and it is offline when $P_{i,t}$ is 0. As stated, $P_{i,t}$ can take any value of $0 \cup [P_{i\min}, P_{i\max}]$, and is a semi-continuous variable.

In Figs. 1(a) and (b), we show the feasible regions of $P_{i,t}$ alone, and $P_{i,t}$ and $u_{i,t}$ coupled, respectively, to simply explain the relation between them. The feasible regions of $P_{i,t}$ are depicted in red dots and red lines, and those of $u_{i,t}$ are plotted in green squares. We can see that the search space becomes more complicated when $P_{i,t}$ is coupled with a binary variable.

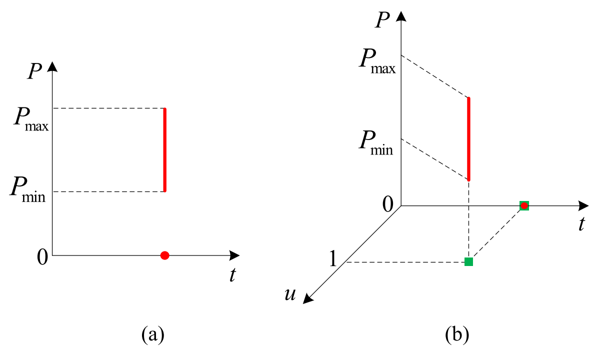

FIGURE 1. Feasible regions. (a) *P* alone. (b) *P* and *u* coupled.

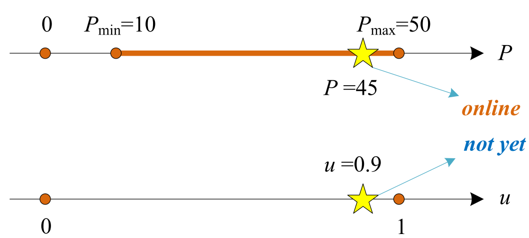

FIGURE 2. Different mechanisms of binary and semi-continuous variables.

Fig. 2 further illustrates the different mechanisms underlying binary and semi-continuous variables in ACUC. For instance, suppose $P_{i\min} = 10$ (MW), $P_{i\max} = 50$ (MW), and we have $u_{i,t} = 0.9$, $P_{i,t} = 45$ (MW) at the time. Although $u_{i,t}$ is close to 1, it is still fractional, thus it needs to undergo further procedures to achieve integrality. However, we note that $P_{i,t}$ is already within its lower and upper limits and needs no further procedure to determine whether the unit is on or off.

To simplify the whole MINLP problem, we reformulate ACUC based on its semi-continuous variables without using auxiliary binary variables.

B. FORMULATING THE CONSTRAINTS AND OBJECTIVE FUNCTION

As the new formulation is mainly meant to deal with $P_{i,t}$ and $u_{i,t}$, the key components are the constraints concerning generator operation, which are presented first and followed by the objective function. The power flow constraints and voltage limits, i.e., constraints (4), (5), (12), and (13), remain unchanged.

1) LOWER AND UPPER LIMITS OF GENERATING UNITS

The lower and upper limits of $P_{i,t}$ are the feasible region of the semi-continuous variable, and the limits of $Q_{i,t}$ should satisfy: a) if the unit is offline ($P_{i,t}$ is zero), then $Q_{i,t}$ is zero; b) if the unit is online ($P_{i,t} \geq P_{i\min}$), then $Q_{i,t}$ is within its limits. We meet these requirements by:

$$P_{i,t} \in 0 \cup [P_{i\min}, P_{i\max}] \qquad i \in \Omega_G,\ t \in \Omega_T \quad (15)$$

$$\begin{cases} Q_{i\min} \leq Q_{i,t} \leq Q_{i\max} \\ -M \cdot P_{i,t} \leq Q_{i,t} \leq M \cdot P_{i,t} \end{cases} \qquad i \in \Omega_G,\ t \in \Omega_T, \quad (16)$$

where M is a large number known as the "big-M." When $P_{i,t} \geq P_{i\min}$, the region of $[-M \cdot P_{i,t},\ M \cdot P_{i,t}]$ includes $[Q_{i\min}, Q_{i\max}]$, thus forcing $Q_{i,t}$ within its original lower and upper limits. Note that $Q_{i\min}$ is assumed to be non-positive here.

In the case that $Q_{i\min}$ is positive for some generators, the interval of $[Q_{i\min}, Q_{i\max}]$ does not include zero thus (16) will be false when the generator is supposed to be offline. Hence, for this kind of generator, $Q_{i,t}$ is also regarded as a semi-continuous variable, and the constraint $Q_{i\min} \leq Q_{i,t} \leq Q_{i\max}$ in (16) should be modified as $Q_{i,t} \in 0 \cup [Q_{i\min}, Q_{i\max}]$.

2) RAMP RATE CONSTRAINTS

To eliminate the binary variables in (8) and (9), we have

$$\begin{cases} P_{i,t} - P_{i,t-1} \leq R_i^U \\ P_{i,t} - P_{i,t-1} \leq P_{i\min} + M \cdot P_{i,t-1} \end{cases} \qquad i \in \Omega_G,\ t \in \Omega_T \quad (17)$$

$$\begin{cases} P_{i,t-1} - P_{i,t} \leq R_i^D \\ P_{i,t-1} - P_{i,t} \leq P_{i\min} + M \cdot P_{i,t} \end{cases} \qquad i \in \Omega_G,\ t \in \Omega_T, \quad (18)$$

where similarly, M is meant to make a unit obey its original ramp-up and ramp-down rate limits when there is no operational state change. To better understand the state changes of a unit that is described by $P_{i,t}$ according to the ramp rate constraints, we demonstrate it in Fig. 3. A unit is started up at time interval t when $P_{i,t-1}$ is zero and $P_{i,t}$ equals $P_{i\min}$; and it is shut down at time interval t when $P_{i,t-1}$ equals $P_{i\min}$ and $P_{i,t}$ is zero, which are the same as the binary-variable-based model.

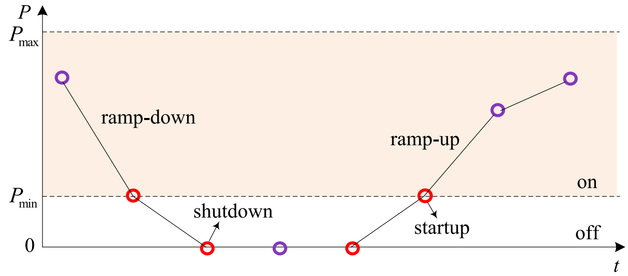

FIGURE 3. State changes described by ***P***.

3) MINIMUM ONLINE AND OFFLINE DURATION TIME CONSTRAINTS

Constraints (10) and (11) can be reformulated as:

$$P_{i,t} \geq P_{i\min}, \quad t \in [1, \max(0, \overline{T}_i - \overline{T}_i^0)], \quad i \in \Omega_G \quad (19a)$$

$$P_{i\min}(P_{i,t} - P_{i,t-1}) \leq P_{i,t} \cdot P_{i,\tau} + M \cdot P_{i,t-1}, \quad \tau \in [t+1, \min(t + \overline{T}_i - 1, N_T)], \quad t \in [\max(0, \overline{T}_i - \overline{T}_i^0) + 1, N_T], \quad i \in \Omega_G. \quad (19b)$$

$$P_{i,t} = 0, \quad t \in [1, \max(0, T_i - T_i^0)], \quad i \in \Omega_G \quad (20a)$$

$$P_{i,t-1} - P_{i,t} \leq P_{i\min} - P_{i,t-1} \cdot P_{i,\tau} + M \cdot P_{i,t}, \quad \tau \in [t+1, \min(t + T_i - 1, N_T)], \quad t \in [\max(0, T_i - T_i^0) + 1, N_T], \quad i \in \Omega_G. \quad (20b)$$

Therefore, when the initial up/down time is less than the minimum up/down time constraints, the unit is forced to be on/off at the beginning of the scheduling horizon. For instance, if $\overline{T}_i = 8$ and $\overline{T}_i^0 = 3$, then $P_{i,t}$ should be no less than $P_{i\min}$ from hour 1 to hour 5 according to (19a), i.e., the unit will be kept online for the first 5 hours. And from hour 6, the unit has to meet the ordinary up/down time limits. For example, when unit i is started up at time interval t, i.e., $P_{i,t-1}$ is zero and $P_{i,t}$ equals $P_{i\min}$, then from (19b), we have that $P_{i,\tau}$ will be no less than $P_{i\min}$ during the following $\overline{T}_i - 1$ time intervals, i.e., the unit will remain online. Moreover, when unit i is shutdown at time interval t, i.e., $P_{i,t-1}$ is $P_{i\min}$ and $P_{i,t}$ equals zero, then from (20b), we have that $P_{i,\tau}$ will be no greater than zero during the following $\overline{T}_i - 1$ time intervals, i.e., the unit will remain offline. And if there is not a state change, $P_{i,\tau}$ can either take any value or the ramp rate constraints are actually active.

4) SPINNING RESERVE CONSTRAINTS

The spinning reserve constraints can be approximated as:

$$\sum_{i \in \Omega_G} [P_{i\max} \cdot \frac{P_{i,t}}{P_{i,t} + \varepsilon} - P_{i,t}] \geq SR_t \quad t \in \Omega_T, \quad (21)$$

where ε is a small positive number close to zero. The first term in the square brackets of (21) is approximately equal to $P_{i\max}$ when $P_{i,t} > 0$.

5) STARTUP COST CONSTRAINTS

The startup cost at time interval t is only incurred when there is a startup state transition, which means $P_{i,t-1}$ must be zero and $P_{i,t}$ is $P_{i\min}$. Thus (2) can be replaced by:

$$C_{i,t}^{su} \geq \frac{\sigma_i}{P_{i\min}}(P_{i,t} - P_{i,t-1} - M \cdot P_{i,t-1}) \quad i \in \Omega_G,\ t \in \Omega_T, \tag{22}$$

combined with

$$C_{i,t}^{su} \geq 0 \quad i \in \Omega_G,\ t \in \Omega_T. \tag{23}$$

The $M \cdot P_{i,t-1}$ part in (22) is to avoid wrongly counting a startup cost when $P_{i,t} > P_{i,t-1} \geq P_{i\min}$, because it can force the right side of (22) to be negative such that (23) will be truly active.

6) OBJECTIVE FUNCTION

Like (21), the objective function can be expressed as:

$$f_{gen}(P_{i,t}) \approx a_i \cdot \frac{P_{i,t}}{P_{i,t} + \varepsilon} + b_i \cdot P_{i,t} + c_i \cdot P_{i,t}^2 + C_{i,t}^{su}, \tag{24}$$

Thus, when $P_{i,t} > 0$, the first term of (24) is very close to a_i.

In summary, the new ACUC formulation with semi-continuous variables is:

$$\min \sum_{i\in\Omega_G} \sum_{i\in\Omega_T} f_{gen}(P_{i,t})$$
$$s.t.\ (4), (5), (12), (13), (15) - (23). \tag{25}$$

C. SETTING OF PARAMETER M

The proposed formulation has some big-M parameters in it, and parameters that are too large can affect the solution of ACUC obtained by a solver. Therefore, we discuss the parameters used in this work.

1) M FOR REACTIVE POWER GENERATION

If $P_{i,t}$ is not zero, then $P_{i,t} \geq P_{i\min}$, due to its semi-continuous character, or $P_{i,t}/P_{i\min} \geq 1$. Thus, for $Q_{i\min} \leq 0$, if we set M in (16) as:

$$\frac{Q_{i\min}}{P_{i\min}} \cdot P_{i,t} \leq Q_{i,t} \leq \frac{Q_{i\max}}{P_{i\min}} \cdot P_{i,t}, \tag{26}$$

then the region of $Q_{i,t}$ constrained by (26) will include $[Q_{i\min}, Q_{i\max}]$ whenever $P_{i,t} \geq P_{i\min}$.

For $Q_{i\min} > 0$, the left side of (26) will be larger than the original $Q_{i\min}$ when the unit is online. In this case, (26) can be set as:

$$\frac{Q_{i\min}}{P_{i\max}} \cdot P_{i,t} \leq Q_{i,t} \leq \frac{Q_{i\max}}{P_{i\min}} \cdot P_{i,t}, \tag{27}$$

where $P_{i,t}/P_{i\max} \leq 1$, thus the left side of (27) will be less than $Q_{i\min}$ but greater than zero when $P_{i,t}$ is positive.

2) M FOR RAMP RATE CONSTRAINTS

Following the idea described above, the values of M in (17) and (18) are set such that the right sides of the constraints will be greater than the original maximum ramp rates:

$$P_{i,t} - P_{i,t-1} \leq P_{i\min} + \frac{R_i^U - P_{i\min}}{P_{i\min}} \cdot P_{i,t-1} \tag{28}$$

$$P_{i,t-1} - P_{i,t} \leq P_{i\min} + \frac{R_i^D - P_{i\min}}{P_{i\min}} \cdot P_{i,t}. \tag{29}$$

3) M FOR MINIMUM ONLINE/OFFLINE DURATION TIME CONSTRAINTS

M can influence these constraints only when a unit remains online, i.e., when $P_{i,t-1}$ and $P_{i,t} \geq P_{i\min}$, and the constraints should not affect the correct ramp rate limits. The maximum value of the left side of (19) can be $P_{i\min}R_i^U$, and the minimum value of $P_{i,t}P_{i,\tau}$ can be zero, thus we need $M \cdot P_{i,t-1} \geq P_{i\min}R_i^U$. Since it is the case when $P_{i,t-1} \geq P_{i\min}$, then M is set as:

$$P_{i\min}(P_{i,t} - P_{i,t-1}) \leq P_{i,t} \cdot P_{i,\tau} + R_i^U \cdot P_{i,t-1}. \tag{30}$$

The maximum value of the left side of (20) can be R_i^D, and the minimum value of $P_{i\min} - P_{i,t-1}P_{i,\tau}$ can be $P_{i\min} - P_{i\max}P_{i\max}$, thus we need $M \cdot P_{i,t} \geq R_i^D + P_{i\max}P_{i\max} - P_{i\min}$. Then M is set as:

$$P_{i,t-1} - P_{i,t} \leq P_{i\min} - P_{i,t-1}P_{i,\tau} + \frac{R_i^D + P_{i\max}P_{i\max} - P_{i\min}}{P_{i\min}} \cdot P_{i,t}. \tag{31}$$

4) M FOR STARTUP COST CONSTRAINTS

Since M is introduced in case $P_{i,t} > P_{i,t-1} \geq P_{i\min}$ for (22) to make its right side negative, and $P_{i,t} - P_{i,t-1}$ is no greater than R_i^U due to the ramp-up rate limit, M can be set as:

$$C_{i,t}^{su} \geq \frac{\sigma_i}{P_{i\min}}(P_{i,t} - P_{i,t-1} - \frac{R_i^U}{P_{i\min}} \cdot P_{i,t-1}). \tag{32}$$

V. CASE STUDIES

We present numerical results on two IEEE test systems and a large-scale system in China to assess the effectiveness of the proposed formulation and method. The problems were solved through GAMS 24.3.3 [27] on a Dell Precision workstation equipped with a 3.50 GHz Intel Xeon CPU E3-1270 and 32 GB RAM.

In addition, N_T was set at 24 intervals and ε was set to 0.01. In the following, the ACUC formulation with binary variables is denoted by B-ACUC, and the new proposed formulation based on semi-continuous variables is denoted by S-ACUC. In GAMS, the B-ACUC and S-ACUC problems were solved by the SBB solver [28], which is based on the standard branch-and-bound (B&B) method, combined with the CONOPT solver [29], which was used to solve the corresponding nonlinear subproblems (which were in fact multi-period AC optimal power flow problems in this work), with default parameters except that the time limit was increased to 20 hours. It is worth mentioning that SBB + CONOPT

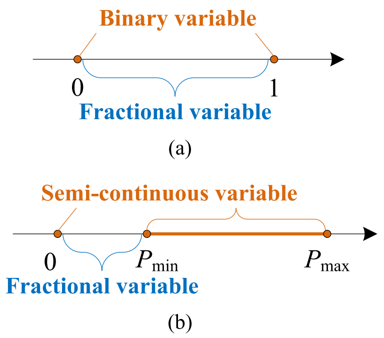

FIGURE 4. Fractional variables. (a) Binary variable. (b) Semi-continuous variable.

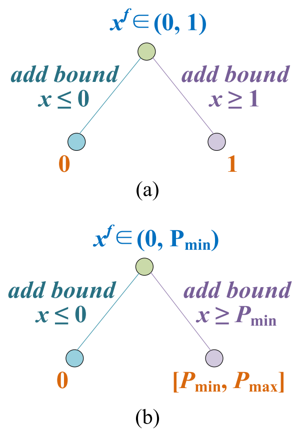

FIGURE 5. Branching and node creation. (a) Binary variable. (b) Semi-continuous variable.

may find a local optimum, but it can provide solutions of the relatively large-scale ACUC problems more effectively. We also tested the BARON solver [30] for B-ACUC, but it hit the time limit in our cases before it could offer a solution.

In addition to CPU time and the objective cost, several indicators generated during the process of B&B were used to evaluate the performance of B-ACUC and S-ACUC:

- *Fractional variables*: According to [19], the number of fractional variables in the continuous relaxation NLP problem can reflect the tightness of a formulation. By solving a relaxed nonlinear programming (NLP) problem of the original B-ACUC or S-ACUC, namely, the binary variables $u_{i,t}$ are relaxed to [0, 1] and the semi-continuous variables $P_{i,t}$ are relaxed to $[0, P_{i\max}]$, fractional solutions are recognized, which belong to (0, 1) for $u_{i,t}$ and $(0, P_{\min})$ for $P_{i,t}$. Fig. 4 explains the definition of fractional variables. And for $u_{i,t}$ or $P_{i,t}$ with fractional values, B&B will perform branching and create two new nodes as illustrated in Fig. 5, during which bounds are added to create new nodes, where x is the variable and x^f demotes the current fractional value. These steps are repeated until the final solution is found. In other words, a smaller number of fractional variables means a larger percentage of binary/semi-continuous solutions for the original variables $u_{i,t}/P_{i,t}$ in the relaxed NLP problem. Thus, a more effective search can be confirmed. We supplemented this feature by providing the number of nodes visited by the SBB solver. It is worth mentioning that fractional variables are defined according to the temporary results of binary or semi-continuous variables in the solution process, and at the end of the calculation, all the fractional variables become binary or semi-continuous.
- *Integrality gap*: Analogous to [18], the integrality gap, defined as $(Z_{FINAL}-Z_{NLP})/Z_{FINAL}$, can measure the tightness of a formulation as well, where Z_{NLP} is the objective value of the original relaxed NLP problem, and Z_{FINAL} is that of the final solution. The reason is that a tighter formulation often provides a larger Z_{NLP} value (strong lower bound) which is closer to Z_{FINAL}, leading to a smaller gap. However, the relaxed NLP problem for ACUC may be trapped in a local optimum by a solver, with the result that the obtained Z_{NLP} is greater than its true value. Also, it is difficult to solve the problem for its global optimum but within an optimal tolerance in practice, thus, the objective costs obtained from the models may differ from each other. Nevertheless, we used the better value obtained from the two formulations to represent Z_{FINAL}, using Z_{NLP} as found by the solver to calculate the gap (locally).

A. RESULTS ON 39-BUS 10-UNIT SYSTEM

To measure the computational performance of S-ACUC and verify the accuracy of the reformulation, we studied this system. The parameters of generators were set as follows: a) $Q_{i\max}$, $Q_{i\min}$, and $P_{i\max}$ were acquired from the website motor.ece.iit.edu/data/data39.xlsx; b) ramp-up and ramp-down rates were set as 70% of $P_{i\max}$; c) $P_{i\min}$, minimum up/down time, and cost coefficients were from [31] corresponding to the sequence of $P_{i\max}$. The normalized daily load profile was also from [31].

TABLE 1. Computational performance of the models in the 39-Bus system.

	B-ACUC	S-ACUC
Cost ($)	2,340,668.90	2,333,438.47
Time (s)	255.02	26.54
int-Gap (%)	2.07	0.059
# of Frac./Disc.	189/240	28/240
# of Nodes	177	29

In Table 1, we summarize the objective cost, CPU time, integrality gap (int-Gap), number of fractional variables along with the total number of discrete variables (# of Frac./Disc.), and number of nodes (# of Nodes) visited by the SBB solver for each model. To maintain a fair comparison, the objective cost of S-ACUC was transferred to the value of the original quadratic production cost, which was calculated after the schedule plan was determined, although the approximation was rather close. The comparisons of these indices are depicted in Fig. 6 in terms of the ratios with respect to the results of B-ACUC, which are always represented as 100%. From Table 1 and Fig. 6, we can observe that the objective

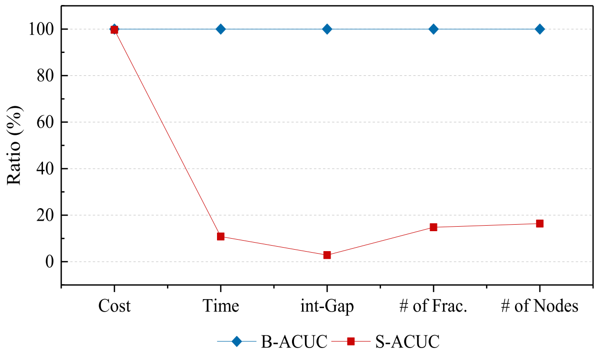

FIGURE 6. Comparisons of computational performance in the 39-bus system. (a) Binary variable. (b) Semi-continuous variable.

costs obtained from B-ACUC and S-ACUC are close to each other, but other mentioned indices of S-ACUC are much smaller and better than those of B-ACUC. Particularly, the number of fractional solutions is 85% fewer, and the CPU time decreases by 90%.

The smaller gap and fewer fractional solution numbers of S-ACUC suggested that it had a tighter characteristic than B-ACUC, thus the search space for the solver to explore for an integer solution was reduced. Besides, the number of nodes visited by SBB reflected the number of NLP problems that were solved, hence the overall problem scale solved for S-ACUC was smaller than for B-ACUC. As a result, S-ACUC could provide a solution in much less time than B-ACUC, which was consistent with the index values.

Fig. 7 displays the generation schedules of active power for S-ACUC, to verify our reformulation. The outputs of all units are plotted in Fig. 7(a). The cost-effective units with higher capacity are scheduled online all day long, providing a large amount of the power to reduce the total generation cost. Other units are scheduled more flexibly, with output power almost following the load curve without violating the operating limits.

Taking unit 4 as an example, the startup and shutdown ramp rate of this unit is 25 MW/h, which is the same as its lower output limit. In Fig. 7(b), we can see that the unit starts up in hour 7 and shuts down in hour 23, so the power outputs in hours 7 and 22 should be 25 MW, which is corroborated by the figure. Moreover, the largest ramp-up rate in Fig. 7(b) is 310.50 MW/h, and the largest ramp-down value is 384.80 MW/h. And the maximum ramp-up and ramp-down rate of this unit is 547.68 MW/h, so the ramping constraints are obeyed.

The minimum up and down time constraints are not obvious from Fig. 7(b), because the unit only starts once due to the comparatively high startup cost. To further verify these constraints, we ignored all the startup costs of the units, hence, a schedule plan with more state changes was obtained. The updated output of unit 4 is depicted in Fig. 8, and we can find that this unit is in startup again in hour 19 after it is shutdown in hour 16. It has been scheduled offline in hours 16, 17, and 18, which meets its minimum down time limit, i.e., 3 hours. Besides, the largest ramp rate as

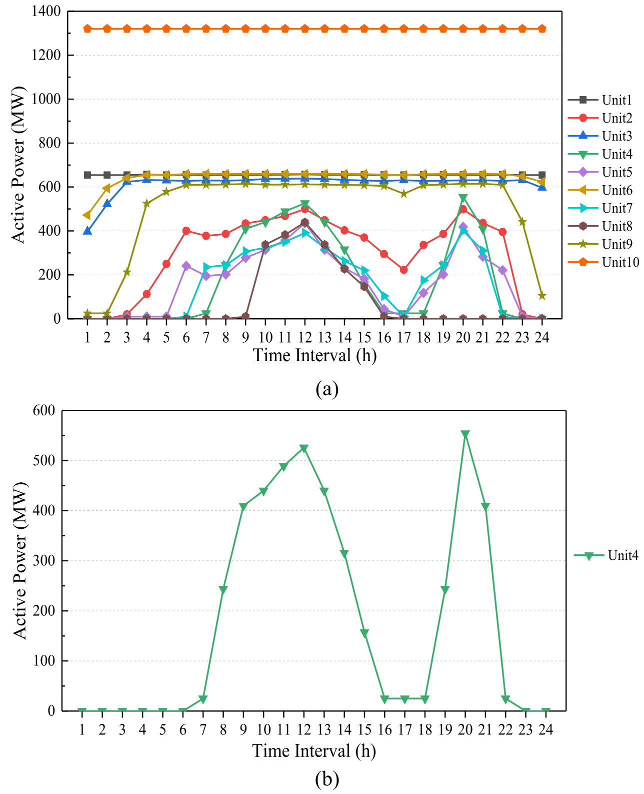

FIGURE 7. Active power generation for S-ACUC in the 39-bus system. (a) Outputs of all units. (b) Output of unit 4.

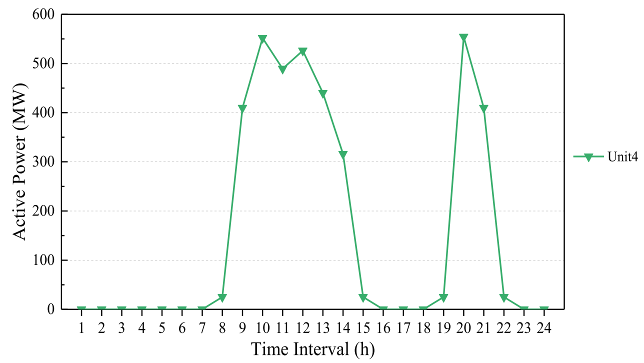

FIGURE 8. Output of unit 4 ignoring startup costs.

shown in Fig. 8 is 529.40 MW/h, which is also within its maximum ramp rate limit.

B. RESULTS ON 118-BUS 54-UNIT SYSTEM

To better test the effectiveness of S-ACUC, we investigated this system with a larger size. The normalized daily load profile was acquired from [32], and the parameters of generators were downloaded from the website motor.ece.iit.edu/data/118_UMP.xls.

The computational performance of the models is listed in Table 2, and the results are plotted in Fig. 9 in terms of ratios. Conclusions similar to those of the 39-bus system can be drawn. S-ACUC was solved quickly by the SBB solver, which reached a similar solution but spent only 22% of the time compared to B-ACUC. Unsurprisingly, the values of the

TABLE 2. Computational performance of the models in 118-Bus system.

	B-ACUC	S-ACUC
Cost ($)	1,188,777.00	1,182,435.09
Time (s)	4,513.07	981.74
int-Gap (%)	0.30	0.12
# of Frac./Disc.	529/1296	144/1296
# of Nodes	464	124

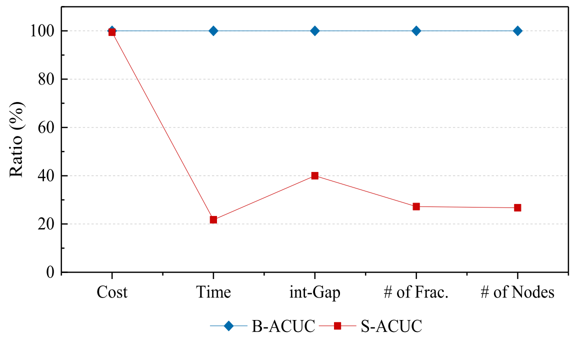

FIGURE 9. Comparison of computational performance in the 118-bus system.

TABLE 3. Computational performance of the models in 739-Bus system.

	B-ACUC	S-ACUC
Cost ($)	–	9,679,818.15
Time (s)	–	25,098.85
int-Gap (%)	–	0.0097
# of Frac./Disc.	–	30/2976
# of Nodes	–	30

integrality gap, number of fractional variables, and number of nodes visited were all smaller than those of B-ACUC. The proposed formulation, S-ACUC, did not lose effectiveness in this system, but still outperformed the original B-ACUC formulation, especially regarding CPU time.

C. RESULTS ON 739-BUS 124-UNIT SYSTEM

To further compare the performance of B-ACUC and S-ACUC, we studied a large-scale test system with 739 nodes, 124 generators, and 867 branches, which was a reduced China Southern Power Grid. Table 3 presents the computational performance of B-ACUC and S-ACUC. From this, we can see that the SBB solver failed to solve B-ACUC within the time limit. By contrast, the problem was successfully solved using S-ACUC. Overall, the S-ACUC model was solved more effectively than by B-ACUC, and the computational performance was significantly improved.

D. COMPARISON WITH DIFFERENT B-ACUC FORMULATIONS

Besides the B-ACUC formulation presented in Section II, which is based on one set of binary variables (1-bin), there are other B-ACUC formulations that use three sets of binary variables (3-bin) or a relaxed version of it (R3-bin), i.e., the binary variables for startup and shutdown changes are relaxed to the continuous interval [0,1] in UC [18]. We compared these 1-bin, 3-bin, R3-bin, and semi-continuous formulations for ACUC to further investigate their computational performance. B-ACUCs all failed to be solved in the large-scale

TABLE 4. Computational performance of different ACUC formulations.

	Items	B-ACUC (1-bin)	B-ACUC (3-bin)	B-ACUC (R3-bin)	S-ACUC
39-bus system	Cost ($)	2,340,668.90	2,335,530.77	2,333,302.31	2,333,438.47
	Time (s)	262.57	1,516.06	256.03	26.54
118-bus system	Cost ($)	1,188,777.00	1,210,800.00	1,195,303.27	1,182,435.09
	Time (s)	4,513.07	26,512.08	20,843.47	981.74

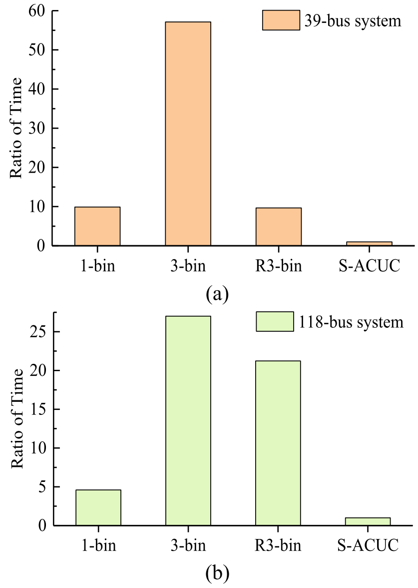

FIGURE 10. Comparison of computational time.

739-bus system. Table 4 summarizes the general results for the 39- and 118-bus test systems obtained by SBB. Fig. 10 displays the ratios of the computational time spent by the B-ACUCs to that of S-ACUC, which is always represented as one.

In Table 4, we note that R3-bin was more effective than 3-bin in these two cases as R3-bin provided lower-cost operation plans in less CPU time than 3-bin. In the 39-bus system, the computing time of 1-bin and R3-bin was almost the same; but in the 118-bus system, 1-bin performed better than R3-bin, which indicated that the R3-bin model was more complex to solve due to the larger number of variables. And the proposed S-ACUC, which provided economical operation plans in these cases and required the least computing time, showed its advantages, especially in the large-scale problem.

VI. CONCLUSIONS

In this work, we propose a new formulation for ACUC based on its inherent semi-continuous variables, namely, the active power outputs of units, with few and close approximations compared to the commonly used binary paradigm. This new formulation directly describes the operation levels and state changes of units through semi-continuous variables, in lieu of auxiliary binary variables to represent the on/off states, thus reducing the number of variables. Comparisons with different ACUC formulations have been investigated. According to the case studies, the proposed formulation is verified and can leverage commercial solvers to obtain a solution more quickly.

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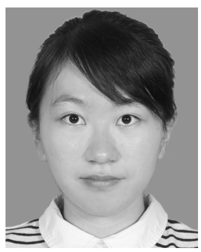

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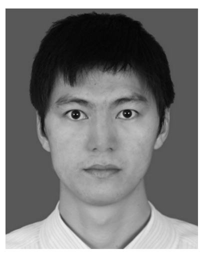

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