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ORIGINAL ARTICLE

Wild horse optimizer: a new meta-heuristic algorithm for solving engineering optimization problems

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Abstract
Nowadays, the design of optimization algorithms is very popular to solve problems in various scientific fields. The optimization algorithms usually inspired by the natural behaviour of an agent, which can be humans, animals, plants, or a physical or chemical agent. Most of the algorithms proposed in the last decade inspired by animal behaviour. In this article, we present a new optimizer algorithm called the wild horse optimizer (WHO), which is inspired by the social life behaviour of wild horses. Horses usually live in groups that include a stallion and several mares and foals. Horses exhibit many behaviours, such as grazing, chasing, dominating, leading, and mating. A fascinating behaviour that distinguishes horses from other animals is the decency of horses. Horse decency behaviour is such that the foals of the horse leave the group before reaching puberty and join other groups. This departure is to prevent the father from mating with the daughter or siblings. The main inspiration for the proposed algorithm is the decency behaviour of the horse. The proposed algorithm was tested on several sets of test functions such as CEC2017 and CEC2019 and compared with popular and new optimization methods. The results showed that the proposed algorithm presented very competitive results compared to other algorithms. The source code is currently available for public from: https://www.mathworks.com/matlabcentral/fileexchange/90787-wild-horse-optimizer.

Keywords Optimization techniques · Meta-heuristic algorithm · Horse algorithm · Wild horse optimizer

1 Introduction

Optimization is the goal of all decision-making problems in all practical fields, from engineering to economics. In optimization theory and its existing methods, it tried to obtain the best case from the set of possible conditions by the specified cost function [1]. Optimization algorithms have become very popular in the last decade. The popularity of these algorithms is due to their simplicity, flexibility, non-inference mechanism and avoidance of local optimal compared to conventional optimization methods. All of these advantages are due to the random nature of the optimization algorithms, which enables them to resolve complex problems with high dimensions at the lowest possible time. Optimization or meta-heuristic algorithms usually inspired by the concepts of biology, animal behaviour, or physics [2]. The algorithms start their work with an initial random population [3]. This population performs the search process in a certain number of iterations [4]. The search process consists of two stages of exploration and exploitation. The difference between the optimization algorithms is in the mechanism used to perform the search and create a balance between exploration and exploitation phases. The genetic algorithm (GA) is one of the oldest and most famous evolutionary algorithms that use the principles of natural selection of Darwinian Theory to find optimal solutions. This algorithm uses genetic evolution as a problem-solving pattern. The problem that must be solved is the input and solutions coded according to a pattern, the fitness function also evaluates each candidate solution. Then, within a stochastic process, a new generation is created. In this process, the concepts of biological science such as inheritance, mutation, natural selection and crossover used [5]. Particle or birds swarm

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optimization (PSO) algorithm is one of the most important algorithms in the field of collective intelligence. This algorithm inspired by the social behaviour of animals such as fish and birds that live together in small and large groups. In particle swarm optimization, all members of the population interact with each other and solve the problem through information exchange [6]. Xin-She Yang released the firefly algorithm (FA) in 2008. He adapted the firefly swarm behaviour to develop an algorithm for optimizing functions with multiple optima. In particular, the concept was used of how the brightness of individual fireflies brings them closer together, and a random factor would be used to encourage the exploration of the solution space [7]. The grey wolf optimizer (GWO) algorithm mimics the leadership hierarchy and the grey wolf hunting mechanism in nature. Four types of grey wolves, alpha, beta, delta, and omega, are used to simulate leadership hierarchies. Also, to simulate the hunting mechanism, three search steps are performed for prey, siege and attack on the prey [8]. The gravitational search algorithm (GSA) is inspired by the gravitational force, which in nature causes the objects to be attracted to each other. The object with more mass has more gravitational and applies it to other objects and absorb fewer weight objects [9]. Ant Lion Optimizer (ALO) is inspired by how ant lions hunt in nature. The hunting process includes five steps of the random movement of ants, making traps, trapping ants in traps, catching prey and rebuilding traps [10]. Whale optimization algorithm (WOA) mimics the social behaviour of humpback whales. The algorithm is inspired by the bubble-net hunting strategy [11]. Multi-verse optimizer (MVO) is based on three concepts in cosmology: white hole, black hole, and wormhole. The mathematical models of these three concepts are developed to perform exploration, exploitation, and local search, respectively [12]. The salp swarm algorithm (SSA) is inspired by the swarm behaviour of salps when navigating and foraging in oceans [13]. The poor and rich optimization (PRO) algorithm is inspired by the efforts of the rich and poor groups to achieve wealth and improve their economic situation. The rich always try to increase their class gap with the poor by gaining wealth from different ways. The rich are always trying to increase their class gap with the poor by acquiring wealth from different ways. On the other hand, the poor try to gain wealth and reduce their class gap with the rich. On the other hand, the poor try to gain wealth and reduce their class gap by modeling the rich [14]. Harris hawks optimizer (HHO) is inspired the cooperative behavior and chasing style of Harris' hawks in nature called surprise pounce. In this intelligent strategy, several hawks cooperatively pounce a prey from different directions in an attempt to surprise it [15]. The fitness-dependent optimizer (FDO) is inspired by the swarming behavior of bees for their reproductive process and collective decision-making. FDO is a particle optimization (PSO) algorithm that updates the search agent position by increasing the velocity (pace) [16]. Artificial electric field algorithm (AEFA) is an intelligently designed artificial system that deals with the purpose of function optimization. AEFA works on the principle of Coulombs' law of electrostatic force and Newton's law of motion [17]. The Levy flight distribution (LFD) algorithm is inspired from the Levy flight random walk for exploring unknown large search spaces [18]. The tunicate swarm algorithm (TSA) imitates jet propulsion and swarm behaviors of tunicates during the navigation and foraging process [19].

2 Need for research

There are some gaps, concerns, and problems within the previous swarm-based optimization methods. For example, the poor and rich algorithm (PRO), which has recently been presented, is designed and configured to perform well on only a few simple and old test functions while failing to solve new and complex test functions such as CEC2017 [20]. Therefore, this algorithm needs to be improved to solve complex problems and functions. The Harris hawk optimization (HHO) algorithm is most focused on solving simple, old, high-dimensional problems while performing poorly on complex problems such as CEC2017 and real-world problems. Therefore, this algorithm needs to be improved to solve complex problems and functions. Well-known algorithms such as the gray wolf optimizer (GWO), the whale optimization algorithm (WOA), and the moth–flame optimization (MFO) [21] have almost the same structure and the only difference is in the search range.

An important question arises here, when there are famous algorithms such as those mentioned above, what is needed to offer and present new algorithms. According to the NFL theorem, no optimization algorithm can solve all optimization problems [22]. The average performance of the optimizers is almost the same. So there are a lot of problems that are not still well solved, notwithstanding the popular optimization algorithms, and offering new algorithms that can solve such problems.

This theory is the motivation of the creation of a new optimization algorithm called wild horse optimizer (WHO) to compete with current algorithms in solving problems. In this article, we present a new optimizer algorithm called the wild horse optimizer, which is inspired by the social life behaviour of wild horses.

The rest of the paper organized as follows: Sect. 3 describes the proposed wild horse optimizer algorithm. The results and discussion of the proposed algorithm, benchmark functions, and real problems are presented in Sect. 4. Finally, Sect. 5 concludes this article and offers suggestions for future work.

3 Wild horse optimizer (WHO)

3.1 Description

Social organization horses divided into two groups of territorial and non-territorial. There are many differences between these two types of organizations in the social grouping, bonding and grazing, mating behaviour, leadership hierarchy, and dominance [23, 24]. In this article, our focus is on non-terrestrial horses. Non-terrestrial horses are herds consisting of stable family groups or harems that include a stallion and one or several mares and offspring. Also, there are single groups, including adult stallions and juvenile horses. Stallions are placed close to mares for communication, and mating may occur at any time. Foals usually start grazing in the first week of life and have more grazing and less rest as they get older. The foals leave their parent groups before puberty, and the male horses join the single groups to mature enough to mate. Female foals join other family groups [25]. This departure is to prevent the father from mating with the daughter or siblings [26]. There have been many studies on the dominance and leadership of non-terrestrial horses. Hierarchy is usually linear, and people within the group tend to interact with people of the same degree and age [27]. Article [28] recorded intergroup dominance in a herd of wild horses during the dry season, with higher-ranking groups having access to a water hole if they wished, while lower-class groups waited for several hours. Leadership is a distinct phenomenon from domination. A leader is an animal which determines the speed and direction of movement of a group of animals [29]. In free-ranging non-territorial horses, the leader of a family group is usually the most dominant mare, and the rest of the group follow in order of decreasing dominance, with the stallion usually a short distance behind the group [30].

In this work, we use group behaviours, grazing, mating, domination and leadership to design a wild horse optimizer and perform optimization of various problems.

3.2 Wild horse optimizer (WHO)

The wild horse optimizer consists of five main steps as follows:

1. creating an initial population and forming horse groups and selecting leaders;
2. grazing and mating of horses;
3. leadership and leading the group by the leader (stallion);
4. exchange and selection of leaders;
5. save the best solution.

3.2.1 Creating an initial population

The basic framework of all optimization algorithms is the same. The algorithm starts with $(\vec{x}) = \{\vec{x}_1, \vec{x}_2, \dots, \vec{x}_n\}$ an initial random population. The target function repeatedly evaluates this random population, and a target value is determined $(\vec{O}) = \{O_1, O_2, \dots, O_n\}$. It also improved by a set of rules that are the core of an optimization technique. Because population-based optimization techniques look for the optimal amount of optimization problems, there is no guarantee of finding a solution in one run. However, with sufficient numbers of random solutions and optimization steps (iteration), the probability of finding the optimal global increases.

First, we divide this initial population into several groups. If N is the number of members of the population, the number of groups is $G = \lceil N \times \mathrm{PS} \rceil$. The PS is the percentage of stallions in the total population that we consider as a control parameter for the proposed algorithm. So we have the leader G (stallion) according to the number of groups, and the remaining members (N–G) are divided equally among these groups. Figure 1 shows an example of this population division. The leader of the groups randomly selected at the beginning of the algorithm, and in the later stages, they are selected based on fitness (the best fitness function) among the members of the group.

Figure 2 shows in more detail how stallions and foals were selected from the original population to form different groups. It should be noted that from the beginning we can create two types of populations of stallions and horses and then form different groups.

3.2.2 Grazing behaviour

As mentioned in the previous section, foals usually spend most of their time grazing around their group. To implement grazing behaviour, we consider the stallion to be the centre of the grazing area, and members of the group search around the centre (graze). We proposed Eq. (1) to simulate grazing behaviour. Equation (1) causes group members to move and search around the leader with a different radius.

$$\bar{X}^j_{i,G} = 2Z\cos(2\pi RZ) \times \left(\mathrm{Stallion}^j - X^j_{i,G}\right) + \mathrm{Stallion}^j, \quad (1)$$

where $X^j_{i,G}$ is the current position of the group member (foal or mare), $\mathrm{Stallion}^j$ is the position of the stallion (group leader), Z is an adaptive mechanism calculated by Eq. (2), R is a uniform random number in the range $[-2, 2]$ that causes The grazing of horses at different angles (360 degrees) of group leader, π is the same as the pi number equal to 3.14, The COS function by combining π and R causes the movement in different radius, and finally $\bar{X}^j_{i,G}$ is the new position of the group member when grazing.

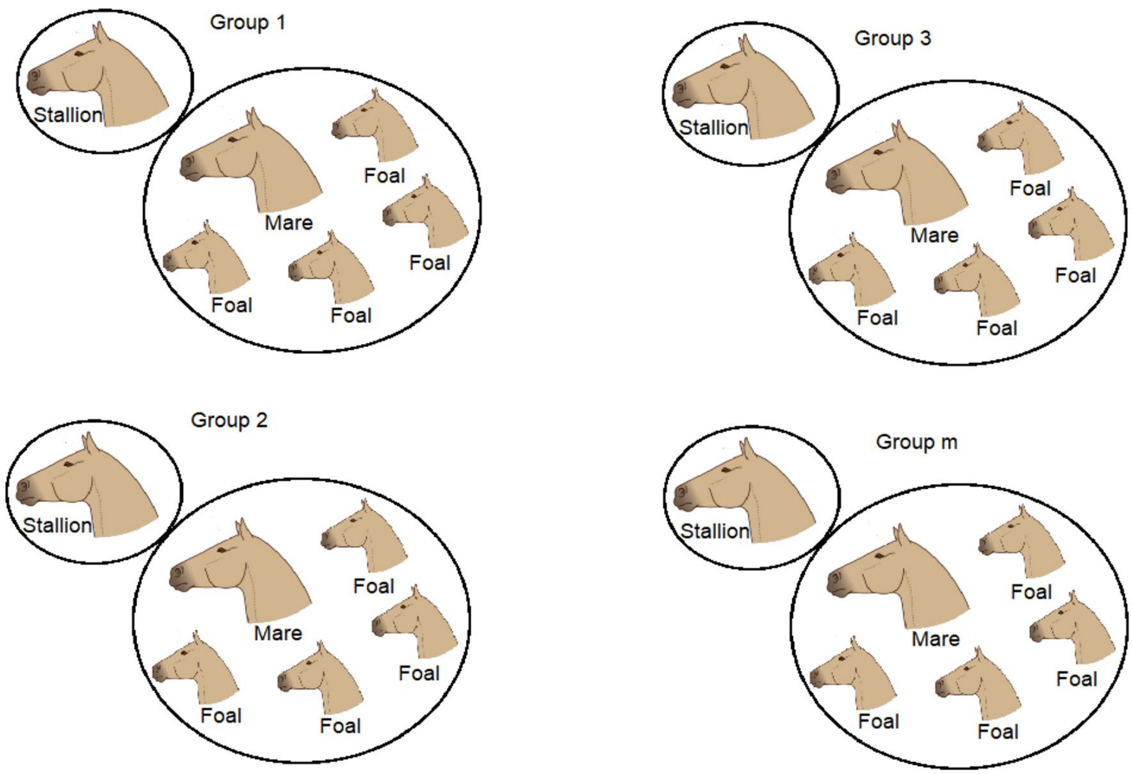
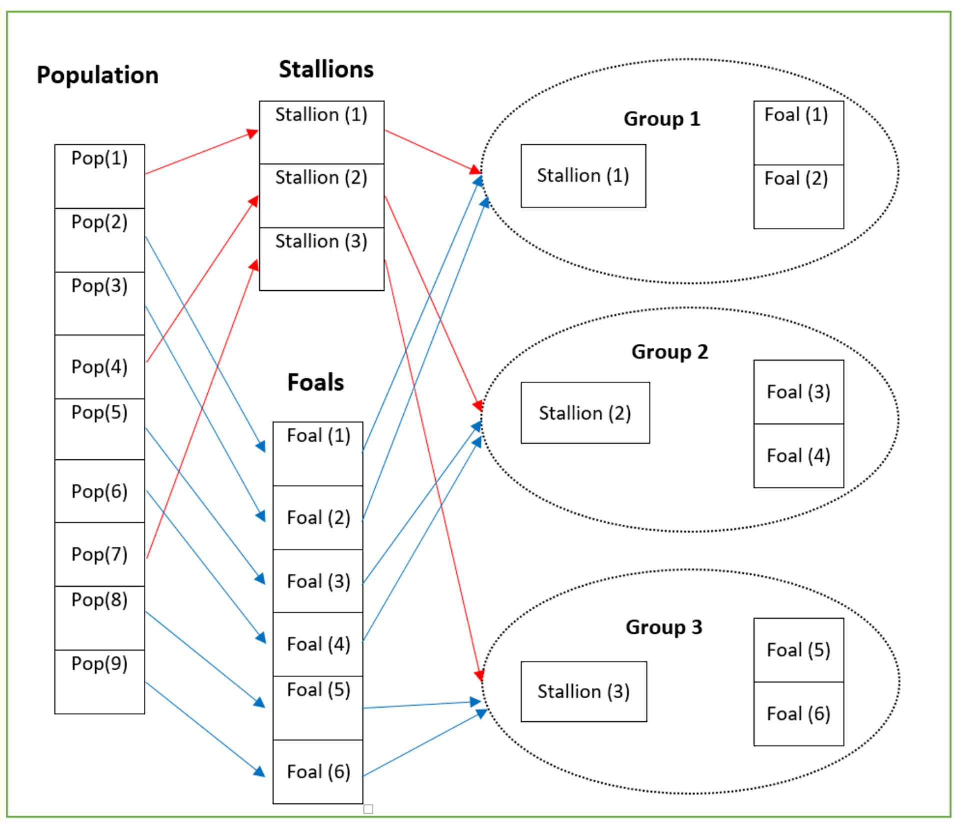


Fig. 1 Formation of horse groups tailored to the initial population

Fig. 2 Formation of groups from orginal population



$$P = \vec{R}_1 < \text{TDR}; \quad IDX = (P == 0); \quad Z = R_2 \Theta \text{IDX} + \vec{R}_3 \Theta (\sim \text{IDX}), \tag{2}$$

where P is a vector consisting of 0 and 1 equal to the dimensions of the problem, $\vec{R}_1$ and $\vec{R}_3$ are random vectors with uniform distribution in the range [0, 1], R_2 is a random number with uniform distribution in the range [0, 1], IDX indexes of the random vector $\vec{R}_1$ returns that satisfy the condition (P == 0). TDR is an adaptive parameter that starts with a value of 1 and decreases during the execution of the algorithm according to Eq. (3) and at the end of the execution of the algorithm reaches 0.

$$\text{TDR} = 1 - \text{iter} \times \left(\frac{1}{\text{maxiter}}\right), \tag{3}$$

where iter is the current iteration and max iter is the maximum number of iterations of the algorithm.

3.2.3 Horse mating behaviour

One of the unique behaviours of horses compared to other animals is the behaviour of separating foals from the group and mating them. Foals leave the group before reaching puberty, and male foals join the group of single horses, and female foals join another family group to reach puberty and find their mate. This departure is to prevent the father from mating with the daughter or siblings. To implement this behaviour, we do the following. A foal leaves group i and joins a temporary group, a foal leaves group j and joins a temporary group. We assume these two foals are male and female, and since these two foals have no family relationship, they can mate after puberty. The resulting child must leave the temporary group and join another group, such as k. This cycle of departure, mating and reproduction is repeated for all different groups of horses. Figure 3 shows this mating and departure behaviour.

To simulate the behaviour of the departure and mating of horses, Eq. (4), which is the same as the Crossover operator of the mean type, has been proposed.

$$X^{p}_{G,K} = \text{Crossover}(X^{q}_{G,i}, X^{z}_{G,j}) \quad i \neq j \neq k, p = q = \text{end},$$
$$\text{Crossover} = \text{Mean} \tag{4}$$

where $X^{p}_{G,K}$ is the position of horse p from group k and leave the group and gives its place to a horse whose parents are horses who have to leave group i and j and have reached puberty. They have no family relationship and have mated and reproduced. $X^{q}_{G,i}$ The position of the foal q is from group i, which his a departure from the group, and after reaching the age of puberty, it mated with the horse z with the position $X^{z}_{G,j}$, which leaves group j.

3.2.4 Group leadership

The group leader must lead the group to a suitable area. We consider this suitable area to be the water hole. The group must move towards this water hole. Other groups move in the same way towards this water hole. Leaders compete for this water hole so that the domination group can use this water hole, and other groups are not allowed to use the water

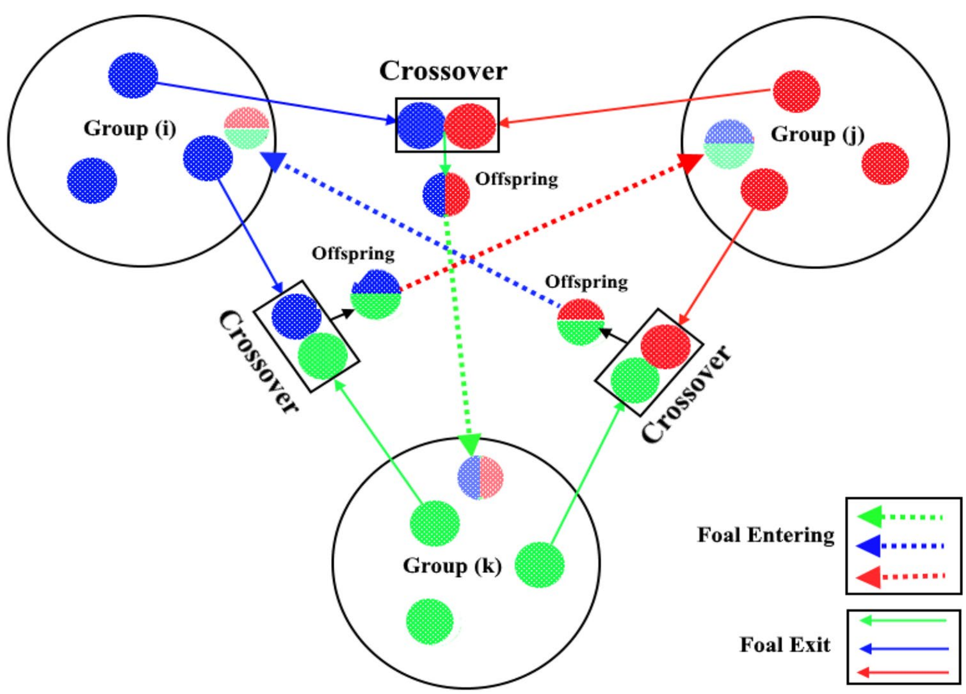


Fig. 3 Departure behaviour of foals out of the group, mating and reproduction

hole until the domination group moves away. The group leaders must lead their group toward this water hole and use the water hole if it dominates, and if another group dominates, they must move away from it. We recommend Eq. (5) to do this approach and distance.

$$\overline{\text{Stallion}_{G_i}} = \begin{cases} 2Z\cos(2\pi RZ) \times (\text{WH} - \text{Stallion}_{G_i}) + \text{WH} & \text{if } R_3 > 0.5 \\ 2Z\cos(2\pi RZ) \times (\text{WH} - \text{Stallion}_{G_i}) - \text{WH} & \text{if } R_3 \leq 0.5 \end{cases}, \quad (5)$$

where $\overline{\text{Stallion}_{G_i}}$ is the next position of the leader of the i group, WH is the position of the water hole, $Stallion_{G_i}$ is the current position of the leader of the i group, Z is an adaptive mechanism calculated by Eq. (2), R is a uniform random number in the range $[-2, 2]$, π is the same as pi number equal to 3.14. Figure 4 shows the update of the $\overline{\text{Stallion}_{G_i}}$ position relative to the best position.

3.2.5 Exchange and selection of leaders

First, we select the leaders randomly to preserve the random nature of the algorithm. In the later stages of the algorithm, leaders are selected based on fitness. If one of the group members is better fitness than the group leader, the position of the leader and the corresponding member will changed according to Eq. (6).

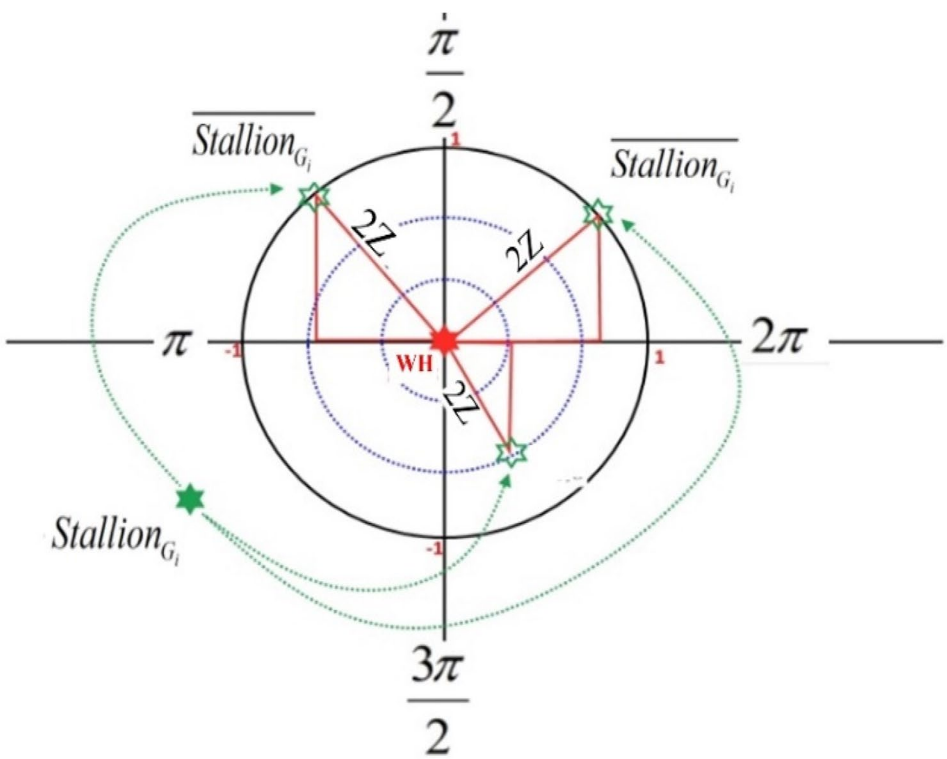
$$\text{Stallion}_{G_i} = \begin{cases} X_{G,i} & \textit{if} \cos t\left(X_{G,i}\right) < \cos t\left(\text{Stallion}_{G_i}\right) \\ \text{Stallion}_{G_i} & \textit{if} \cos t\left(X_{G,i}\right) > \cos t\left(\text{Stallion}_{G_i}\right) \end{cases}. \quad (6)$$

The flowchart of the proposed wild horse optimizer algorithm is shown in Fig. 5 and the pseudo-code is presented in Fig. 6.

3.3 Hypotheses in the horse algorithm

- Exploring the search space is ensured by the random selection of leaders and the random movements of foals around them.
- Due to the departure of the horse from the group and the mating with other horses of other groups, there is a high probability of solving local optimal static.
- Wild horse optimizer is a population-based algorithm, so local optimal avoidance is inherently very high.
- Calculating random movements for each horse and each dimension enhances population diversity. Leaders, during optimization, are moved to the location of the best

Fig. 4 Update of the stallion position relative to the best position



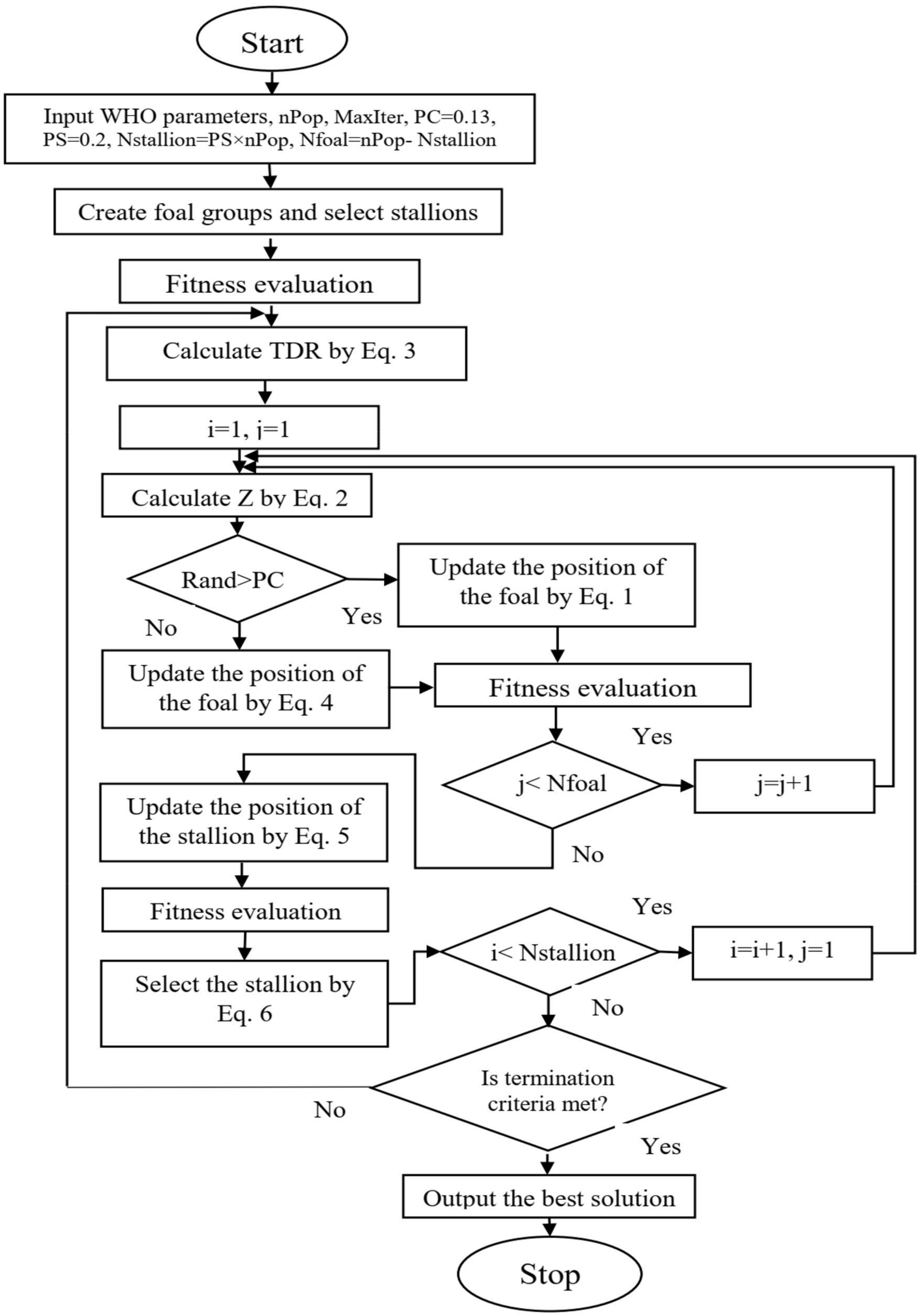


Fig. 5 Flowchart of the proposed WHO algorithm

```
Initialize the first population of Horses randomly
Input WHO parameters,  PC=0.13, PS=0.2
Calculate the fitness of Horses
Create Foal groups and select Stallions
Find the best Horse as the optimum
While the end criterion is not satisfied
Calculate TDR by Eq. 3
 For number of Stallions
    Calculate Z by Eq. 2
      For number of Foals of any group
         If rand>PC
           Update the position of the Foal by Eq. 1
         Else
           Update the position of the Foal by Eq. 4
         End
      End
    If rand>0.5
     Update the position of the StallionGi (overline) by Eq. 5.1
    Else
     Update the position of the StallionGi (overline) by Eq. 5.2
    End

    If cost(StallionGi (overline)) < cost(Stallion)
      Stallion=StallionGi (overline)
    End
    sort Foals of group by cost
    select Foal with Minimum cost
    If  cost(Foal) < cost(Stallion)
      Exchange Foal and Stallion Position by Eq.6
    End
 End
Update optimum
End
```

Fig. 6 Pseudo-code of the wild horse optimizer algorithm

horses and, therefore, the desired areas of the search space are stored.

- Leaders lead the horses toward the desired areas of the search space.
- The best leader is saved in every iteration and compared to the best leader obtained by that moment (optimal).
- The wild horse optimizer algorithm has very few parameters to adjust.
- The wild horse optimizer algorithm is a gradient-free algorithm that considers the problem as a black box.

In the following sections, several sets of test functions and real problems are used to evaluate and validate the performance of the wild horse optimizer algorithm in solving optimization problems.

4 Results and discussion

In this section, several sets of test functions including classical test functions, CEC2017 test functions and CEC2019 test functions with different characteristics and some real problems are proposed to evaluate the performance of the algorithm. The test functions are unimodal, multimodal, hybrid and composition. The following describes the details of these functions and problems.

4.1 Classical test functions

Classic test functions have been used by many researchers [31, 32]. Details of the unimodal (F1–F7) and multimodal (F8–F13) test functions are given in Table 1. Typically, all optimization algorithms have two phases of exploration and exploitation. A unimodal test function has no local optimal, and there is only one global optimal. All search space is dedicated to global optimization. Therefore, the convergence speed and exploitation of an algorithm can be a criterion. Multimodal and hybrid test functions have many local optimal that make them very suitable for evaluating the performance of the algorithm in terms of avoiding local optimal and exploration.

Thirty search agents and 500 iterations with a maximum of 15,000 number function evaluation (NFE) used to solve the test functions. Each of the test functions solved 30 times to generate statistical results. Different performance indicators used to compare the algorithms: best, worst, mean quantitatively, and standard deviation are the best solutions obtained in the last iterations. The value of standard and mean deviations should be less, the ability of an algorithm to avoid local optimal and determine the optimal global is measured. Eight well-known and new algorithms have used to prove the results: particle swarm algorithm (PSO) [6], genetic algorithm (GA) [5], Levy flight distribution (LFD) [18], tunicate swarm algorithm (TSA) [19], gray wolf algorithm (GWO) [8], salp swarm algorithm (SSA) [13], artificial electricity field algorithm (AEFA) [17], and multi-verse optimization (MVO) algorithm [12]. The initial controlling parameters of all algorithms shown in Table 2.

The results of Table 3 show that the proposed algorithm performed better in unimodal functions than other compared algorithms. This accuracy is due to the grazing phase of the group members around the group leader. In multimodal functions, the performance of the proposed algorithm is better than other algorithms. The reason for the excellent performance of the proposed algorithm in the phase of

Table 1 Unimodal-multimodal benchmark functions

Function	Dim	Range	f_{MIN}
$f_1(x) = \sum_{i=1}^{n} x_i^2$	30	[− 100, 100]	0
$f_2(x) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	30	[− 10, 10]	0
$f_3(x) = \sum_{i=1}^{n} \left(\sum_{j-1}^{i} x_j \right)^2$	30	[− 100, 100]	0
$f_4(x) = \max \left\{ \lvert x_i \rvert, 1 \le i \le n \right\}$	30	[− 100, 100]	0
$f_5(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	30	[− 30, 30]	0
$f_6(x) = \sum_{i=1}^{n} \left([x_i + 0.5] \right)^2$	30	[− 100, 100]	0
$f_7(x) = \max \left\{ \lvert x_i \rvert, 1 \le i \le n \right\}$	30	[− 1.28, 1.28]	0
$F_8(x) = \sum_{i=1}^{n} -x_i \sin(\sqrt{\lvert x_i \rvert})$	30	[− 500, 500]	0
$F_9(x) = \sum_{i=1}^{n} [x_i^2 - 10\cos(2\pi x_i) + 10]$	30	[− 5.12, 5.12]	0
$F_{10}(x) = -20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}) - \exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)) + 20 + e$	30	[− 32, 32]	0
$F_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}}) + 1$	30	[− 600, 600]	0
$F_{12}(x) = \frac{\pi}{n}\left\{ 10\sin(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10\sin^2(\pi y_{i+1})] + (y_n - 1)^2 \right\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4) + \sum_{i=1}^{n} u(x_i, 10, 100, 4) \quad y_i = 1 + \frac{x_i + 1}{4}$ $u(x_i, a, k, m) = \left\{ \begin{array}{cc} k(x_i - a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i < -a \end{array} \right\}$	30	[− 50, 50]	0
$F_{13}(x) = 0.1\{\sin^2(3\pi x_1) + \sum_{i=1}^{n}(x_i - 1)^2[1 + \sin^2(3\pi x_i + 1)] + (x_n - 1)^2[1 + \sin^2(2\pi x_n)]\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$	30	[− 50, 50]	0

Table 2 Initial values for the controlling parameters of algorithms

Algorithm	Parameter	Value
WHO	Crossover percentage	PC = 0.13
	Stallions percentage (number of groups)	PS = 0.2
	Crossover	Mean
TSA	Parameter P_{min}	1
	Parameter P_{max}	4
LFD	Threshold	2
	csv	0.5
	β	1.5
	α_1, α_2, α_3	10, 0.0005, 0.005
	d_1, d_2	0.9, 0.1
GA	Type	Real coded
	Selection	Roulette wheel
	Crossover	Single point 0.8
	Mutation	0.3
PSO	Topology	Fully connected
	Cognitive and social constants	C1 = 2, c2 = 2
	Inertial weight	Linearly decreases from 0.9 to 0.4
SSA	Leader position update probability	0.5
AEFA	FCheck	1
	Rpower	1
	Tag	1
	Rnorm	2
FDO	weightFactor	0.0
GSA	Rnorm, Rpower, alpha, and G0	2, 1, 20, 100
GWO	Convergence parameter (a)	Linear reduction from 2 to 0
MVO	WEP_Max, WEP_Min	1, 0.2

TABLE 3 Results for the unimodal and multimodal benchmark functions with 30 dimension and 15,000 NFE

Function		PSO	GA	LFD	MVO	SSA	TSA	AEFA	GWO	WHO
F1	Min	6.3687E−08	1.1862E+00	1.7396E−07	6.7380E−01	3.5349E−08	2.2692E−24	1.9809E−23	7.9298E−29	**1.2954E−51**
	Max	1.7511E−04	9.7369E+00	6.5472E−07	2.5243E+00	1.5906E−06	2.1123E−20	4.5461E+01	1.1401E−26	**9.5089E−43**
	Avg	7.5711E−06	3.7445E+00	3.1894E−07	1.2397E+00	2.1652E−07	1.1952E−21	5.8259E+00	1.5774E−27	**3.7368E−44**
	Std	3.1733E−05	1.8718E+00	9.9187E−08	3.7200E−01	3.5844E−07	3.8106E−21	1.1261E+01	2.3526E−27	**1.7395E−43**
F2	Min	5.2108E−05	3.0150E−01	2.3820E−04	4.2430E−01	2.1000E−03	4.4765E−15	2.5110E+00	2.3844E−17	**1.4153E−28**
	Max	3.5420E−01	6.3210E−01	4.9180E−04	1.2125E+02	5.2057E+00	6.4050E−13	4.0798E+01	6.1818E−16	**6.3278E−23**
	Avg	1.7600E−02	4.3610E−01	3.3826E−04	1.2701E+01	1.8670E+00	1.2037E−13	1.6421E+01	1.1513E−16	**3.4738E−24**
	Std	6.4200E−02	8.7300E−02	5.9285E−05	3.3916E+01	1.3512E+00	1.6792E−13	9.9241E+00	1.1772E−16	**1.3164E−23**
F3	Min	5.9600E+01	2.7777E+03	5.9159E−07	9.6760E+01	2.6928E+02	7.8777E−08	7.2897E+02	7.5275E−09	**1.8914E−34**
	Max	4.0824E+02	9.6350E+03	2.6905E−06	4.6326E+02	3.3176E+03	8.9000E−03	4.1889E+03	1.4831E−05	**6.6070E−24**
	Avg	1.5136E+02	4.9062E+03	1.5075E−06	2.2397E+02	1.5842E+03	7.3622E−04	2.1335E+03	3.6567E−06	**2.9866E−25**
	Std	8.4138E+01	1.6773E+03	5.2652E−07	8.2725E+01	9.7086E+02	2.2000E−03	7.9890E+02	4.5551E−06	**1.2374E−24**
F4	Min	7.9560E−01	5.0016E+00	2.3536E−04	8.2220E−01	5.6891E+00	1.6000E−03	2.4332E+00	6.2193E−08	**8.3901E−21**
	Max	4.9876E+00	1.4062E+01	4.7033E−04	4.3259E+00	1.8129E+01	2.2713E+00	9.2042E+00	3.0803E−06	**1.0320E−15**
	Avg	2.6037E+00	8.6089E+00	3.5503E−04	2.0770E+00	1.0729E+01	3.4200E−01	5.7974E+00	5.4883E−07	**5.1113E−17**
	Std	8.1480E−01	2.2117E+00	5.7888E−05	7.2820E−01	3.0387E+00	4.5530E−01	1.5690E+00	6.2983E−07	**1.9068E−16**
F5	Min	**1.3574E+01**	1.2802E+02	2.7720E+01	3.2401E+01	2.2800E+01	2.7121E+01	2.0030E+02	2.5846E+01	2.5454E+01
	Max	1.5323E+02	1.2819E+03	2.8232E+01	2.4821E+03	1.2098E+04	2.8893E+01	2.4035E+04	**2.8748E+01**	8.2087E+01
	Avg	4.8076E+01	3.8267E+02	2.8045E+01	3.7717E+02	6.4378E+02	2.8538E+01	5.6236E+03	**2.7054E+01**	2.8849E+01
	Std	3.4486E+01	2.1821E+02	1.4330E−01	6.1918E+02	2.1859E+03	4.9750E−01	6.1974E+03	**7.6380E−01**	1.0135E+01
F6	Min	9.0877E−08	1.5493E+00	**1.2894E+00**	4.8280E−01	3.8656E−08	2.0882E+00	2.1626E−05	2.3020E−01	7.5856E−06
	Max	1.5284E−05	1.2629E+01	2.2355E+00	1.9342E+00	**4.4351E−06**	4.7821E+00	6.0482E+01	1.6613E+00	3.2890E−01
	Avg	1.8869E−06	4.4623E+00	1.8794E+00	1.3054E+00	**2.7368E−07**	3.7085E+00	5.6147E+00	7.7600E−01	1.4200E−02
	Std	2.9636E−06	2.6180E+00	2.3630E−01	3.9120E−01	**7.9301E−07**	5.9560E−01	1.3061E+01	3.9630E−01	6.0100E−02
F7	Min	1.2100E−02	4.5000E−02	2.0770E−01	1.5400E−02	6.4300E−02	5.9000E−03	7.7400E−02	5.1470E−04	**5.1684E−05**
	Max	4.0800E−02	2.4890E−01	2.8580E+00	7.7800E−02	2.8710E−01	2.2900E−02	1.7180E+00	4.4000E−03	**9.6000E−03**
	Avg	2.3800E−02	1.3020E−01	1.0666E+00	3.6800E−02	1.6520E−01	1.1500E−02	4.1830E−01	1.8000E−03	**1.3000E−03**
	Std	8.2000E−03	4.0400E−02	6.3480E−01	1.5600E−02	6.1900E−02	4.2000E−03	3.4850E−01	1.1000E−03	**1.7000E−03**
F8	Min	−7.5142E+03	−1.1367E+04	−5.1390E+03	−8.8729E+03	−8.9091E+03	−7.7656E+03	−3.3903E+03	−7.4057E+03	−1.0491E+04
	Max	−4.3174E+03	−1.0178E+04	−3.5453E+03	−6.4853E+03	−5.8071E+03	−4.5518E+03	−1.9866E+03	−3.3270E+03	−7.9397E+03
	Avg	−6.1610E+03	−1.0784E+04	−4.0790E+03	−7.6158E+03	−7.2944E+03	−5.9703E+03	−2.6471E+03	−5.8179E+03	−9.0247E+03
	Std	8.0918E+02	3.5453E+02	3.4111E+02	5.5407E+02	7.6897E+02	6.7836E+02	4.3489E+02	8.3600E+02	6.5132E+02
F9	Min	2.1889E+01	3.1102E+00	1.2260E−06	6.0122E+01	1.0945E+01	9.9417E+01	1.8957E+01	5.6843E−14	**0.0000E+00**
	Max	1.1044E+02	1.3078E+01	1.3355E−05	1.5609E+02	1.0049E+02	3.3722E+02	5.6713E+01	1.8792E+01	**3.3866E−07**
	Avg	4.8454E+01	7.3239E+00	4.4945E−06	1.1284E+02	5.3297E+01	1.8795E+02	3.7298E+01	2.9046E+00	**1.3694E−08**
	Std	2.1084E+01	2.1164E+00	2.8664E−06	2.5162E+01	1.8520E+01	5.1117E+01	9.2125E+00	4.2140E+00	**6.2773E−08**

TABLE 3 (continued)

Function		PSO	GA	LFD	MVO	SSA	TSA	AEFA	GWO	WHO
F10	Min	9.4966E−05	2.3210E−01	8.1467E−05	6.1430E−01	1.1551E+00	1.5854E−12	3.1131E−12	6.4837E−14	**8.8818E−16**
	Max	2.3162E+00	1.7192E+00	1.6892E−04	1.9531E+01	3.7343E+00	3.7062E+00	4.3799E+00	1.3589E−13	**4.4409E−15**
	Avg	1.1436E+00	7.8710E−01	1.3374E−04	2.3397E+00	2.3167E+00	1.0001E+00	1.6697E+00	1.0036E−13	**1.5987E−15**
	Std	8.1430E−01	4.0360E−01	1.9538E−05	3.2863E+00	7.2420E−01	1.5589E+00	1.0125E+00	1.6321E−14	**1.4454E−15**
F11	Min	9.8043E−08	8.5140E−01	4.7386E−07	6.0800E−01	2.0325E−04	0.0000E+00	2.9762E+00	0.0000E+00	**0.0000E+00**
	Max	2.0510E−01	1.0672E+00	1.3466E−06	1.0044E+00	5.0500E−02	2.1800E−02	2.0498E+01	3.1200E−02	**0.0000E+00**
	Avg	2.8100E−02	1.0159E+00	8.2287E−07	8.4950E−01	1.8200E−02	8.8000E−03	9.1667E+00	5.1000E−03	**0.0000E+00**
	Std	4.1900E−02	5.3700E−02	2.3613E−07	9.1700E−02	1.2500E−02	8.3000E−03	3.7706E+00	9.0000E−03	**0.0000E+00**
F12	Min	**1.0676E−08**	3.6000E−03	4.6790E−01	3.9720E−01	1.2302E+00	6.7580E−01	1.1729E+00	1.2800E−02	1.0591E−06
	Max	5.1910E−01	2.2710E−01	9.8600E−01	5.4374E+00	1.8729E+01	1.8139E+01	1.1280E+01	1.1340E−01	**1.0420E−01**
	Avg	5.8900E−02	3.5100E−02	7.8150E−01	2.2890E+00	6.6205E+00	8.2361E+00	4.3879E+00	4.5100E−02	**1.0600E−02**
	Std	1.1140E−01	4.8600E−02	1.3410E−01	1.2256E+00	3.6634E+00	4.6965E+00	2.4032E+00	2.6100E−02	**3.1600E−02**
F13	Min	**6.3977E−08**	1.3740E−01	2.8702E+00	6.9400E−02	7.2000E−03	1.8630E+00	4.2519E+00	1.0860E−01	4.6828E−06
	Max	**2.2250E−01**	7.2350E−01	2.9661E+00	5.1730E−01	4.9103E+01	3.8258E+00	4.5920E+01	1.0581E+00	1.6370E−01
	Avg	**1.8500E−02**	3.5800E−01	2.9599E+00	1.8060E−01	1.5824E+01	2.8779E+00	2.7848E+01	6.3080E−01	3.4000E−02
	Std	**4.6300E−02**	1.3920E−01	2.3500E−02	1.0990E−01	1.4868E+01	5.0150E−01	8.4241E+00	2.1030E−01	4.4000E−02

The bold numbers in a table are the best result that shown in each row of the table

exploration is the movement and guidance of groups to different places by leaders.

Figure 7 shows the qualitative results for evaluating the convergence of the proposed algorithm in different functions. As shown in Fig. 7, the proposed algorithm in unimodal functions follows a specific pattern that gives more importance to the exploitation phase (functions F1 and F3). In multimodal functions that have many local optimal, the proposed algorithm follows a different pattern. It pays more attention to the exploration phase that performed at the early stages of the algorithm. However, in the final stages of the algorithm, which is usually the exploitation phase, the exploration is performed broken form (functions F12 and F13). Almost in all functions, the proposed algorithm has a better convergence pattern.

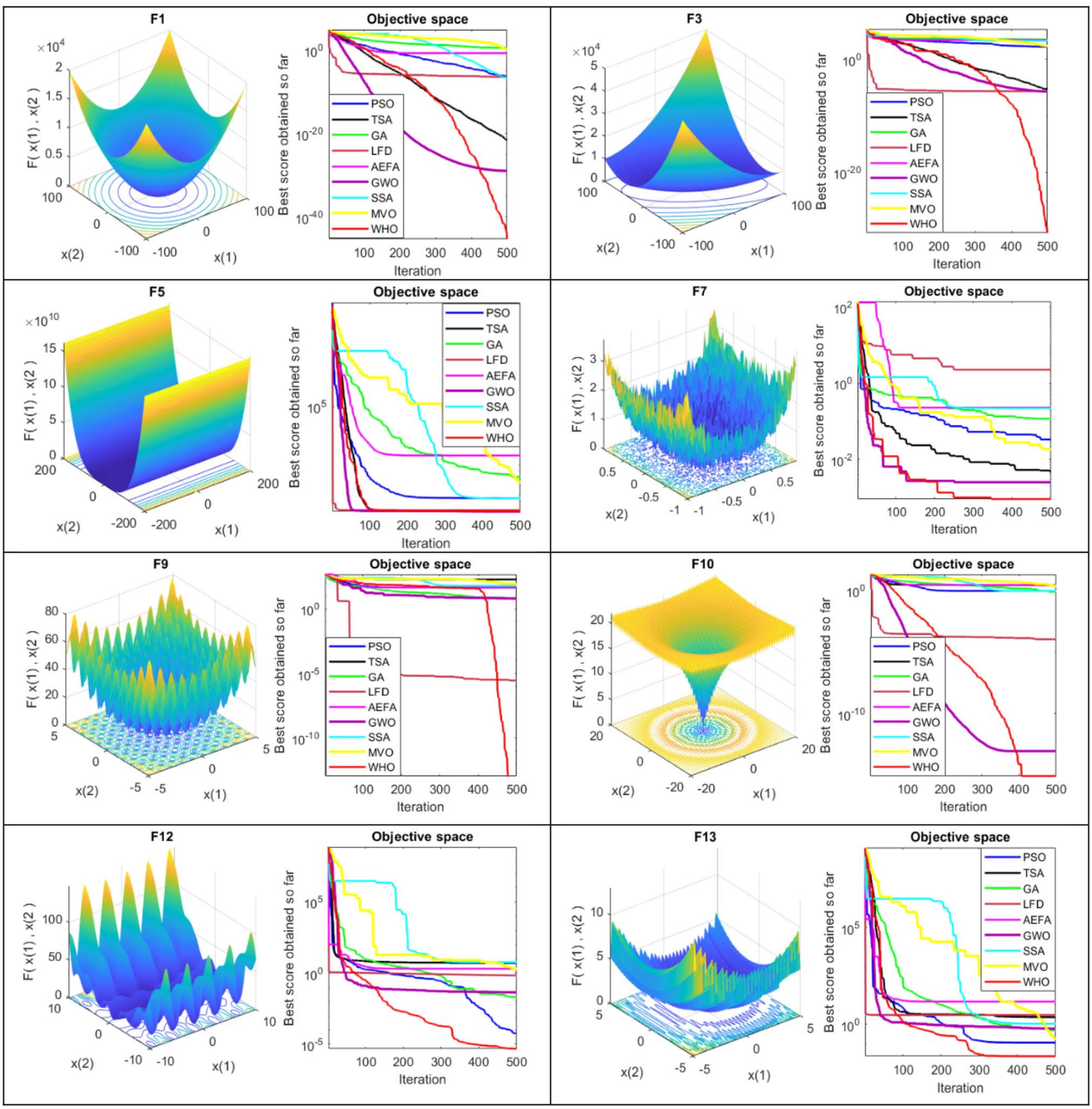


Fig. 7 Convergence curve of algorithms in different functions

4.2 Scalability of wild horse optimizer

In this section, we evaluate the performance of the proposed algorithm in large-scale problems using scalability analysis. We selected three unimodal functions and three multimodal functions to evaluate scalability and adjusted these functions to different dimensions of 10, 30, 50, and 100 dimensions. The results of the scalability evaluation of the proposed algorithm compared to other algorithms are shown in Fig. 8. These results indicate that with increasing the dimensions of the problem in different functions, the performance of the proposed algorithm has not changed much. In contrast, other compared algorithms have shown different performance with increasing dimensions of the problem and their performance has become weaker. The scalability of the proposed algorithm is due to the adaptive parameter Z, which presented in Eq. (2).

4.3 Sensitivity analysis

The proposed WHO algorithm employs four parameters, i.e., number of horses, maximum number of iterations, parameter PC (crossover percentage), and parameter PS (stallions percentage or number of groups). The sensitivity investigation of these parameters has been discussed by varying their values and keeping other parameters fixed as mentioned in Sect. 3.1 and Table 2.

1. *Number of horses*: WHO algorithm was simulate for different values of horse (i.e., 30, 50, 80, 100, 150). Figure 9a show the variations of different number of search

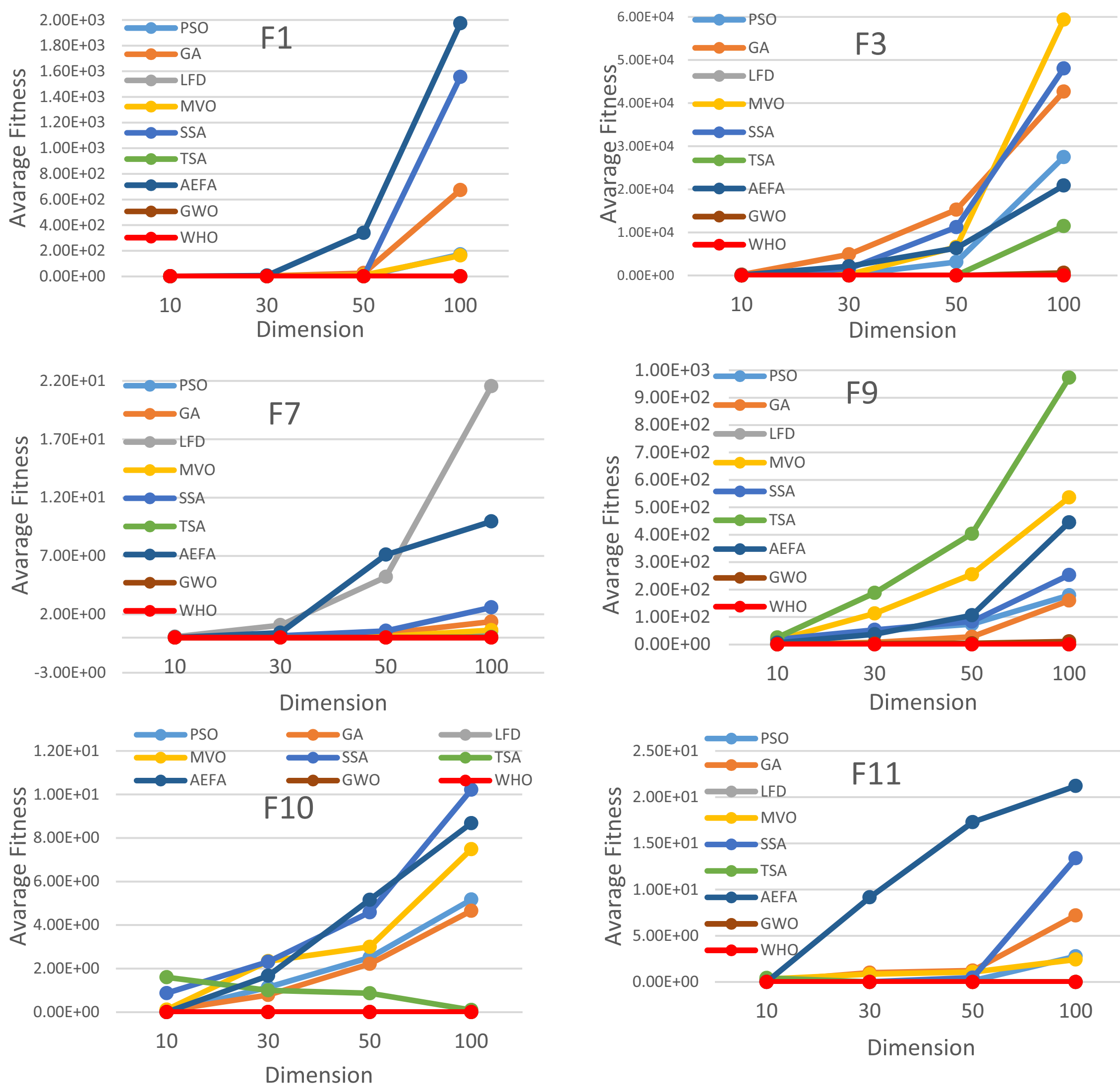


Fig. 8 Scalability of the proposed algorithm

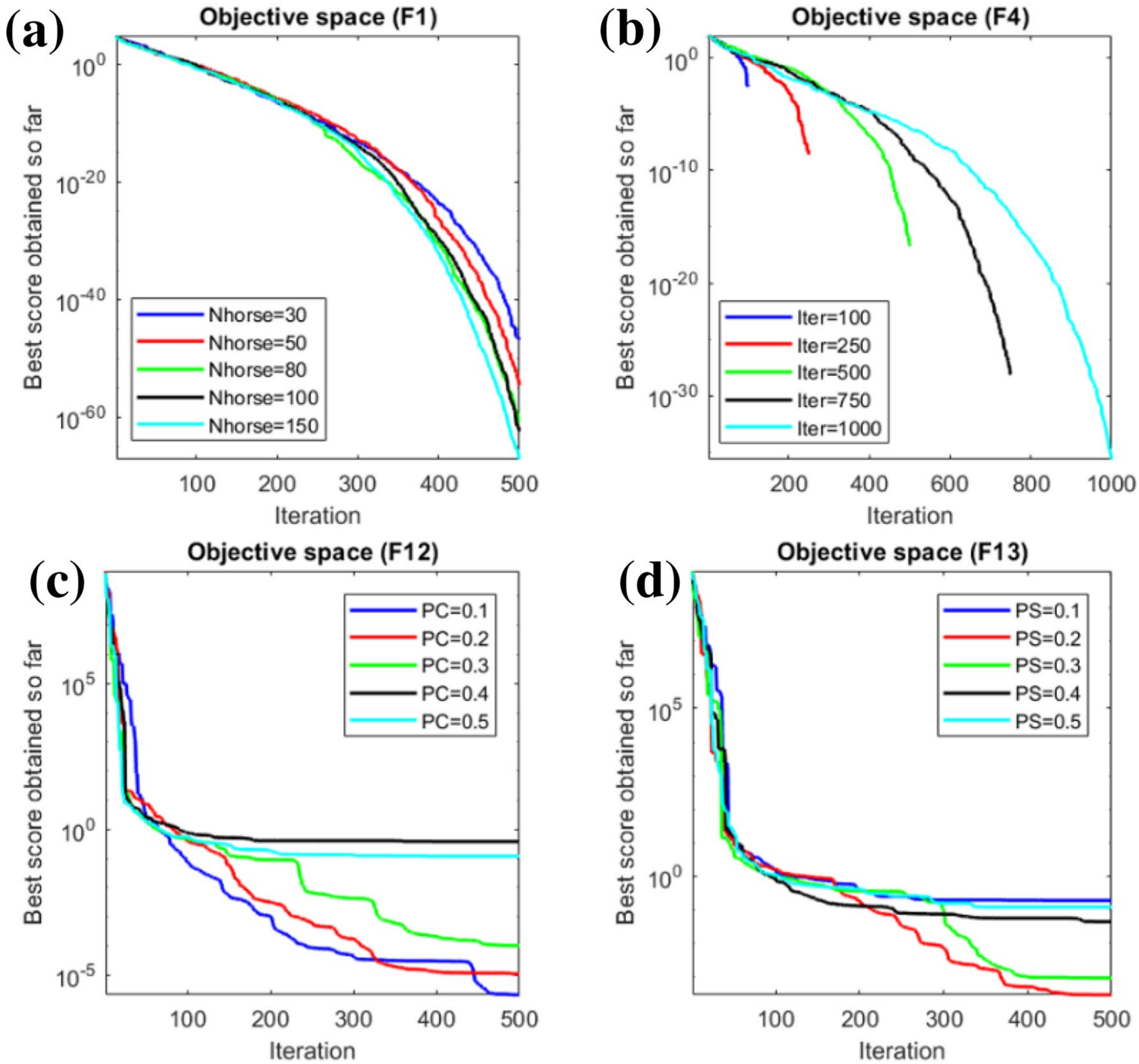


Fig. 9 Sensitivity analysis of proposed WHO algorithm for, **a** number of horses; **b** number of iterations; **c** control parameter PC; **d** control parameter PS

agents on benchmark test functions. It is analyzed from Fig. 9a that the value of fitness function decreases when number of search agents increases.

2. *Maximum number of iterations*: WHO algorithm was run for different number of iterations. The values of $Max_{iteration}$ used in experimentation are 100, 250, 500, 750, and 1000. Figure 9b show the effect of number of iterations over benchmark test functions. The results show that WHO converges towards the optimum when the number of iterations is increased.
3. Variation in parameter PC: To investigate the effect of parameter PC, WHO algorithm was run for different values of PC keeping other parameters fixed. The values of PC used in experimentation are 0.1, 0.2, 0.3, 0.4, and 0.5. Figure 9c show the variation of PC on benchmark test functions. The results show that WHO generates better optimal results when the value of PC is set to 0.1.
4. *Variation in parameter PS*: To investigate the effect of parameter PS, WHO algorithm was run for different values of PS keeping other parameters fixed. The values of PS used in experimentation are 0.1, 0.2, 0.3, 0.4, and 0.5. Figure 9d show the variation of PS on benchmark test functions. The results show that WHO generates better optimal results when the value of PC is set to 0.2.

4.4 The performance of the wild horse optimizer on the CEC2017

To further evaluate the performance of the proposed algorithm, we have selected one of the most challenging test functions called CEC2017, which includes 30 unimodal, multimodal, hybrid and composition test functions [33]. Details of these functions given in Table 4.

We tested the proposed algorithm on CEC2017 test functions and compared with particle swarm optimization (PSO), genetic algorithm (GA), multi-verse optimizer (MVO), whale optimizer algorithm (WOA), salp swarm algorithm (SSA), gray wolf optimizer (GWO), fitness-dependent optimizer (FDO), Harris hawk optimizer (HHO), artificial electric field algorithm (AEFA) and poor and rich optimizer (PRO). The dimensions of all functions are considered 10. Each method tested 30 times with 1000 iterations and a maximum of 60,000 number function evaluation (NFE). We divided the functions into three groups: unimodal–multimodal, hybrid and composition, and compared them separately. The results of unimodal and multimodal functions given in Table 5. Algorithms were ranked using Friedman's mean rank method. The results showed that the proposed algorithm ranked first in solving unimodal and multimodal problems. It is noteworthy that in most functions, the proposed algorithm was able to find the closest global optimal value.

The results of the second group of CEC2017 functions, which are hybrid, are shown in Table 6. The results show

Table 4 Properties and summary of the CEC-BC-2017 test functions (Dim = 10 for all functions)

Type	No.	Functions	Global min	Domain
Unimodal function	F1	Shifted and Rotated Bent Cigar Function	100	[− 100, 100]
	F2	Shifted and Rotated Sum of Different Power Function	200	[− 100, 100]
	F3	Shifted and Rotated Zakharov Function	300	[− 100, 100]
Multimodal functions	F4	Shifted and Rotated Rosenbrock's Function	400	[− 100, 100]
	F5	Shifted and Rotated Rastrigin's Function	500	[− 100, 100]
	F6	Shifted and Rotated Expanded Schaffer's Function	600	[− 100, 100]
	F7	Shifted and Rotated Lunacek Bi_Rastrigin Function	700	[− 100, 100]
	F8	Shifted and Rotated Non-Continuous Rastrigin's Function	800	[− 100, 100]
	F9	Shifted and Rotated Levy Function	900	[− 100, 100]
	F10	Shifted and Rotated Schwefel's Function	1000	[− 100, 100]
Hybrid functions	F11	Hybrid Function of Zakharov, Rosenbrock and Rastrigin's	1100	[− 100, 100]
	F12	Hybrid Function of High Conditioned Elliptic, Modified Schwefel and Bent Cigar	1200	[− 100, 100]
	F13	Hybrid Function of Bent Ciagr, Rosenbrock and Lunache Bi-Rastrigin	1300	[− 100, 100]
	F14	Hybrid Function of Elliptic, Ackley, Schaffer and Rastrigin	1400	[− 100, 100]
	F15	Hybrid Function of Bent Cigar, HGBat, Rastrigin and Rosenbrock	1500	[− 100, 100]
	F16	Hybrid Function of Expanded Schaffer, HGBat, Rosenbrock and Modified Schwefel	1600	[− 100, 100]
	F17	Hybrid Function of Katsuura, Ackley, Expanded Griewank plus Rosenbrock, Modifed Schwefel and Rastrigin	1700	[− 100, 100]
	F18	Hybrid Function of high conditioned Elliptic, Ackley, Rastrigin, HGBat and Discus	1800	[− 100, 100]
	F19	Hybrid Function of Bent Cigar, Rastrigin, Expanded Grienwank plus Rosenbrock, Weierstrass and expanded Schaffer	1900	[− 100, 100]
	F20	Hybrid Function of Happycat, Katsuura, Ackley, Rastrigin, Modified Schwefel and Schaffer	2000	[− 100, 100]
Composition functions	F21	Composition Function of Rosenbrock, High Conditioned Elliptic and Rastrigin	2100	[− 100, 100]
	F22	Composition Function of Rastrigin's, Griewank's and Modified Schwefel's	2200	[− 100, 100]
	F23	Composition Function of Rosenbrock, Ackley, Modified Schwefel and Rastrigin	2300	[− 100, 100]
	F24	Composition Function of Ackley, High Conditioned Elliptic, Girewank and Rastrigin	2400	[− 100, 100]
	F25	Composition Function of Rastrigin, Happycat, Ackley, Discus and Rosenbrock	2500	[− 100, 100]
	F26	Composition Function of Expanded Scaffer, Modified Schwefel, Griewank, Rosenbrock and Rastrigin	2600	[− 100, 100]
	F27	Composition Function of HGBat, Rastrigin, Modified Schwefel, Bent-Cigar, High Conditioned Elliptic and Expanded Schaffer	2700	[− 100, 100]
	F28	Composition Function of Ackley, Griewank, Discus, Rosenbrock, HappyCat, Expanded Schaffer	2800	[− 100, 100]
	F29	Composition Function of shifted and rotated Rastrigin, Expanded Schaffer and Lunacek Bi_Rastrigin	2900	[− 100, 100]
	F30	Composition Function of shifted and rotated Rastrigin, Non-Continuous Rastrigin and Levy Function	3000	[− 100, 100]

Table 5 Results for the CEC-2017 (unimodal–multimodal) test functions with 10 dimension and 60,000 NFE

Function		PSO	GA	MVO	WOA	SSA	GWO	FDO	HHO	AEFA	PRO	WHO
F1	Min	1.0123E+02	3.8536E+03	1.0419E+03	4.1831E+04	1.0119E+02	5.8779E+03	1.8353E+10	9.0647E+04	1.0059E+02	1.4193E+09	**1.0000E+02**
	Max	8.7292E+03	5.9100E+04	1.3532E+04	1.6890E+07	1.0784E+04	4.8796E+08	2.4374E+10	6.7488E+05	3.2795E+03	2.4104E+10	**1.0095E+02**
	Avg	1.8522E+03	2.0762E+04	6.0177E+03	2.3848E+06	3.4884E+03	4.2080E+07	2.1193E+10	3.6428E+05	7.2899E+02	1.0538E+10	**1.0004E+02**
	Std	2.2564E+03	1.3608E+04	3.7938E+03	4.2092E+06	3.2973E+03	1.2229E+08	1.6865E+09	1.5316E+05	8.6122E+02	6.0015E+09	**1.7180E−01**
	Rank	3.0000E+00	6	5	8	4	9	11	7	2	10	1
F2	Min	2.0000E+02	2.0000E+02	2.0000E+02	7.2400E+02	2.0000E+02	3.7200E+02	1.6273E+15	2.0000E+02	5.3950E+03	1.3590E+07	**2.0000E+02**
	Max	2.0000E+02	5.8280E+03	3.4900E+02	1.3673E+06	6.3010E+03	4.3170E+08	2.1086E+17	2.8585E+04	1.3272E+07	7.5384E+12	**2.0000E+02**
	Avg	2.0000E+02	4.2203E+02	2.6563E+02	1.1229E+05	8.0537E+02	1.8217E+07	3.4892E+16	2.3271E+03	1.5058E+06	4.3669E+11	**2.0000E+02**
	Std	0.0000E+00	1.0258E+03	5.2623E+01	2.7282E+05	1.3382E+03	7.8899E+07	5.5312E+16	5.7032E+03	2.6786E+06	1.4330E+12	**0.0000E+00**
	Rank	1	4	3	7	5	9	11	6	8	10	1
F3	Min	3.0000E+02	7.7182E+02	3.0000E+02	3.8188E+02	3.0000E+02	3.2015E+02	1.4575E+04	3.0026E+02	3.9274E+03	6.0074E+03	**3.0000E+02**
	Max	3.0000E+02	7.5502E+03	3.0004E+02	3.3418E+03	3.0000E+02	6.0975E+03	1.5603E+04	3.0513E+02	2.4478E+04	2.1323E+04	**3.0000E+02**
	Avg	3.0000E+02	2.6707E+03	3.0002E+02	1.3395E+03	3.0000E+02	9.0548E+02	1.5382E+04	3.0144E+02	1.2635E+04	1.4714E+04	**3.0000E+02**
	Std	3.5009E−14	1.4643E+03	8.5000E−03	8.7978E+02	7.1720E−10	1.1534E+03	2.6047E+02	1.0172E+00	4.3286E+03	4.0434E+03	**7.0809E−14**
	Rank	1	8	4	7	3	6	11	5	9	10	2
F4	Min	4.0182E+02	**4.0004E+02**	4.0061E+02	4.0237E+02	4.0106E+02	4.0334E+02	2.9835E+03	4.0015E+02	4.0653E+02	4.7104E+02	4.0007E+02
	Max	4.0555E+02	4.6088E+02	4.0555E+02	5.7918E+02	4.0846E+02	4.6339E+02	4.0852E+03	4.7273E+02	4.0738E+02	1.8624E+03	**4.0260E+02**
	Avg	4.0366E+02	4.0761E+02	4.0359E+02	4.4254E+02	4.0472E+02	4.1190E+02	3.4589E+03	4.0996E+02	4.0701E+02	1.0642E+03	**4.0168E+02**
	Std	8.2670E−01	1.0302E+01	1.3375E+00	5.0969E+01	1.4350E+00	1.2681E+01	2.5124E+02	1.6826E+01	2.2100E−01	4.4837E+02	**6.2780E−01**
	Rank	3	6	2	9	4	8	11	7	5	10	1
F5	Min	5.1393E+02	5.0398E+02	5.0597E+02	5.1901E+02	5.0696E+02	5.0554E+02	6.2514E+02	5.1109E+02	**5.0000E+02**	5.4718E+02	5.0199E+02
	Max	5.5870E+02	5.2787E+02	5.3633E+02	5.9358E+02	5.4577E+02	5.2250E+02	6.4675E+02	5.8248E+02	**5.0796E+02**	6.3272E+02	5.1890E+02
	Avg	5.3356E+02	5.1340E+02	5.1570E+02	5.5443E+02	5.1988E+02	5.1230E+02	6.3446E+02	5.4651E+02	**5.0388E+02**	5.9317E+02	5.0866E+02
	Std	1.0662E+01	5.2949E+00	7.5284E+00	2.1295E+01	9.5117E+00	4.9879E+00	5.6416E+00	1.7142E+01	**1.7980E+00**	2.2386E+01	3.5840E+00
	Rank	7	4	5	9	6	3	11	8	1	10	2
F6	Min	6.0000E+02	6.0001E+02	6.0004E+02	6.1237E+02	6.0070E+02	6.0003E+02	6.5334E+02	6.0616E+02	6.0000E+02	6.2357E+02	**6.0000E+02**
	Max	6.3169E+02	6.0011E+02	6.0530E+02	6.5198E+02	6.2721E+02	6.0095E+02	6.5963E+02	6.5950E+02	**6.0008E+02**	6.7967E+02	6.0050E+02
	Avg	6.0707E+02	6.0004E+02	6.0135E+02	6.3116E+02	6.1112E+02	6.0022E+02	6.5734E+02	6.2856E+02	**6.0000E+02**	6.5248E+02	6.0003E+02
	Std	7.6948E+00	2.6800E−02	1.5834E+00	1.2320E+01	7.1241E+00	2.4320E−01	1.3673E+00	1.4009E+01	**1.4200E−02**	1.3835E+01	1.0900E−01
	Rank	6	3	5	9	7	4	11	8	1	10	2
F7	Min	7.1574E+02	7.1353E+02	7.0965E+02	7.3194E+02	7.1526E+02	7.1518E+02	7.9383E+02	7.3820E+02	7.1091E+02	7.6680E+02	**7.0563E+02**
	Max	7.4408E+02	7.3085E+02	7.4754E+02	8.3407E+02	7.6217E+02	7.4907E+02	8.2443E+02	8.2847E+02	**7.1548E+02**	8.7572E+02	7.2492E+02
	Avg	7.2265E+02	7.2070E+02	7.2337E+02	7.8205E+02	7.3361E+02	7.2861E+02	8.0841E+02	7.8643E+02	**7.1255E+02**	8.3207E+02	7.1914E+02
	Std	4.7989E+00	4.5840E+00	8.1829E+00	2.5672E+01	1.1364E+01	9.4036E+00	8.1592E+00	2.1534E+01	**1.0539E+00**	2.4593E+01	3.8106E+00
	Rank	4	3	5	8	7	6	10	9	1	11	2

Table 5 (continued)

Function		PSO	GA	MVO	WOA	SSA	GWO	FDO	HHO	AEFA	PRO	WHO
F8	Min	8.0497E+02	8.0398E+02	8.0697E+02	8.1796E+02	8.0895E+02	8.0601E+02	8.3783E+02	8.1596E+02	**8.0199E+02**	8.2887E+02	8.0298E+02
	Max	8.2786E+02	8.2189E+02	8.3980E+02	8.7074E+02	8.4278E+02	8.3224E+02	8.4215E+02	8.5090E+02	**8.0597E+02**	8.9442E+02	8.1691E+02
	Avg	8.1625E+02	8.1085E+02	8.2053E+02	8.3978E+02	8.2365E+02	8.1353E+02	8.4041E+02	8.2908E+02	**8.0308E+02**	8.6574E+02	8.0693E+02
	Std	5.9545E+00	5.1486E+00	9.7420E+00	1.4798E+01	8.8131E+00	6.9170E+00	1.7563E+00	9.1952E+00	**1.0565E+00**	1.3576E+01	3.3815E+00
	Rank	5	3	6	9	7	4	10	8	1	11	2
F9	Min	9.0000E+02	9.0010E+02	9.0000E+02	1.0189E+03	9.0000E+02	9.0001E+02	1.4963E+03	9.7862E+02	**9.0000E+02**	1.2860E+03	9.0000E+02
	Max	9.0000E+02	9.2200E+02	9.0100E+02	2.2176E+03	1.5993E+03	9.1466E+02	1.7256E+03	1.8097E+03	**9.0000E+02**	2.5244E+03	9.0218E+02
	Avg	9.0000E+02	9.0388E+02	9.0013E+02	1.3577E+03	9.4897E+02	9.0242E+02	1.6531E+03	1.3995E+03	**9.0000E+02**	1.9267E+03	9.0024E+02
	Std	2.9856E−14	4.7266E+00	2.7660E−01	3.1348E+02	1.6801E+02	4.4276E+00	4.2672E+01	2.4732E+02	**0.0000E+00**	3.6513E+02	5.6260E−01
	Rank	2	6	3	8	7	5	10	9	1	11	4
F10	Min	1.1301E+03	**1.0153E+03**	1.1221E+03	1.5416E+03	1.2486E+03	1.1389E+03	3.2893E+03	1.5430E+03	1.2438E+03	2.2669E+03	1.1640E+03
	Max	2.1534E+03	2.1314E+03	2.3338E+03	2.7792E+03	2.4078E+03	2.1561E+03	3.8842E+03	2.5329E+03	2.8477E+03	3.2791E+03	**1.8099E+03**
	Avg	1.8274E+03	1.6696E+03	1.6419E+03	2.0621E+03	1.7979E+03	1.5761E+03	3.6081E+03	1.9859E+03	2.2083E+03	2.7202E+03	**1.4756E+03**
	Std	2.3720E+02	2.4954E+02	3.0413E+02	2.7461E+02	2.5011E+02	2.4399E+02	1.7189E+02	2.7546E+02	3.5861E+02	2.7311E+02	**1.7937E+02**
	Rank	6	4	3	8	5	2	11	7	9	10	1
Friedman mean rank		3.55	4.27	3.73	7.45	5.00	5.10	9.73	6.73	3.45	9.36	**1.64**
Rank		3	5	4	9	6	7	11	8	2	10	**1**

The bold numbers in a table are the best result that shown in each row of the table

Table 6 Results for the CEC-2017 hybrid functions test functions with 10 dimension and 60,000 NFE

Function		PSO	GA	MVO	WOA	SSA	GWO	FDO	HHO	AEFA	PRO	WHO
F11	Min	1.1050E+03	1.1067E+03	1.1087E+03	1.1141E+03	1.1087E+03	1.1037E+03	1.3097E+04	1.1088E+03	1.1576E+03	1.1463E+03	**1.1010E + 03**
	Max	1.1777E+03	1.1580E+03	1.3172E+03	1.3068E+03	1.3412E+03	1.2351E+03	1.5406E+07	1.3441E+03	2.3025E+03	8.9123E+03	**1.1757E + 03**
	Avg	1.1334E+03	1.1247E+03	1.1309E+03	1.1830E+03	1.1755E+03	1.1333E+03	2.4642E+06	1.1531E+03	1.4636E+03	1.9271E+03	**1.1090E+03**
	Std	1.9642E+01	1.1423E+01	4.2390E+01	4.7035E+01	6.3279E+01	3.0304E+01	3.8727E+06	4.4932E+01	3.1607E+02	1.6295E+03	**1.5881E+01**
	Rank	5	2	3	9	8	4	11	7	6	10	1
F12	Min	2.0205E+03	2.1461e+04	3.9000e+03	6.3301e+03	3.3009e+04	8.7028e+03	1.5793e+09	1.1657e+04	2.4996e+04	8.6435e+05	**1.3609e+03**
	Max	5.2710E+04	5.4094e+06	3.5143e+06	1.8585e+07	6.2454e+06	2.1446e+06	3.5300e+09	1.3491e+07	6.5886e+06	2.3892e+09	**6.1652e+04**
	Avg	1.3683E+04	1.3533e+06	5.3338e+05	3.4375e+06	1.5039e+06	6.3458e+05	2.3930e+09	3.3976e+06	1.2298e+06	3.6208e+08	**8.6424e+03**
	Std	1.0944E+04	1.3556e+06	8.4417e+05	4.1109e+06	1.6403e+06	6.9823e+05	5.7508e+08	3.8744e+06	1.4033e+06	7.1105e+08	**1.1761e+04**
	Rank	2	6	3	9	7	4	11	8	5	10	1
F13	Min	1.6661E+03	1.3059e+03	1.3734e+03	1.6190e+03	2.0493e+03	2.5849e+03	3.9113e+07	1.7725e+03	5.4356e+03	1.5348e+03	**1.3069e+03**
	Max	1.6708E+04	1.9639e+04	3.0310e+04	4.2251e+04	5.3400e+04	2.8156e+04	1.7261e+09	3.5436e+04	1.9869e+04	1.7852e+04	**2.5673e+03**
	Avg	8.9503E+03	8.0309e+03	9.1541e+03	1.4944e+04	1.5195e+04	1.1651e+04	4.8419e+08	1.6127e+04	1.1418e+04	5.4672e+03	**1.5149e+03**
	Std	4.3964E+03	4.7297e+03	8.9011e+03	1.0909e+04	1.2601e+04	6.1889e+03	4.3172e+08	1.1318e+04	3.8107e+03	4.2611e+03	**3.3442E+02**
	Rank	4	3	5	8	9	7	11	10	6	2	1
F14	Min	1.4398E+03	1.4491E+03	1.4242E+03	1.4694E+03	1.4550E+03	1.4463E+03	5.5888E+05	1.4707E+03	1.7752E+03	1.4383E+03	**1.4050E+03**
	Max	3.3246E+03	1.8554E+04	1.4593E+03	4.9379E+03	1.6381E+03	5.5832E+03	1.1218E+09	2.3042E+03	1.0376E+04	1.5769E+03	**1.4849E+03**
	Avg	1.8370E+03	4.4663E+03	1.4348E+03	1.7320E+03	1.5042E+03	3.0668E+03	3.0406E+08	1.5539E+03	4.4929E+03	1.4960E+03	**1.4327E+03**
	Std	5.5978E+02	3.7949E+03	8.6277E+00	6.3983E+02	3.8309E+01	1.8532E+03	2.8026E+08	1.4642E+02	2.1203E+03	3.9275E+01	**1.9121E+01**
	Rank	7	9	2	6	4	8	11	5	10	3	1
F15	Min	1.5055E+03	1.5266E+03	1.5067E+03	1.6760E+03	1.6276E+03	1.5681E+03	3.3354E+04	1.6997E+03	4.5919E+03	1.6853E+03	**1.5015E+03**
	Max	4.0262E+03	1.6762E+04	1.5875E+03	1.5163E+04	4.7408E+03	6.1031E+03	9.6601E+07	6.1607E+03	3.1681E+04	8.4837E+03	**1.6726E+03**
	Avg	1.8305E+03	4.3549E+03	1.5410E+03	5.0172E+03	2.3984E+03	3.0216E+03	1.0541E+07	3.0773E+03	1.2982E+04	3.0035E+03	**1.5334E+03**
	Std	5.4564E+02	3.9794E+03	2.4010E+01	3.3603E+03	7.2658E+02	1.3161E+03	2.0198E+07	1.2867E+03	4.8366E+03	1.8137E+03	**4.5838E+01**
	Rank	3	8	2	9	4	6	11	7	10	5	1
F16	Min	1.6005E+03	1.6011E+03	1.6021E+03	1.6121E+03	1.6051E+03	1.6040E+03	2.2734E+03	1.6195E+03	1.8512E+03	1.7561E+03	**1.6002E+03**
	Max	2.0558E+03	1.9756E+03	2.0124E+03	2.1051E+03	1.9044E+03	2.0122E+03	2.5718E+03	2.0786E+03	2.2091E+03	2.6070E+03	**1.9892E+03**
	Avg	1.8521E+03	1.7821E+03	1.7384E+03	1.8422E+03	1.7133E+03	1.6841E+03	2.4006E+03	1.8823E+03	2.0254E+03	2.0884E+03	**1.6688E+03**
	Std	1.3169E+02	1.0627E+02	1.1457E+02	1.3159E+02	8.0336E+01	8.7073E+01	7.2087E+01	1.2058E+02	9.6303E+01	1.6841E+02	**9.3620E+01**
	Rank	7	5	4	6	3	2	11	8	9	10	1
F17	Min	1.7236E+03	1.7027E+03	1.7240E+03	1.7379E+03	1.7225E+03	1.7167E+03	1.7999E+03	1.7276E+03	1.7510E+03	1.7552E+03	**1.7018E+03**
	Max	1.8642E+03	1.7605E+03	1.8628E+03	1.8897E+03	1.9215E+03	1.8100E+03	2.2134E+03	1.8736E+03	2.0144E+03	2.0418E+03	1.7417E+03
	Avg	1.7541E+03	**1.7204E+03**	1.7720E+03	1.7897E+03	1.7734E+03	1.7461E+03	1.9350E+03	1.7797E+03	1.8088E+03	1.8765E+03	1.7233E+03
	Std	3.1020E+01	**1.2973E+01**	4.5570E+01	4.3582E+01	4.3034E+01	1.8972E+01	1.0178E+02	3.5434E+01	6.5194E+01	1.0283E+02	1.0568E+01
	Rank	4	1	5	8	6	3	11	7	9	10	2

Table 6 (continued)

Function		PSO	GA	MVO	WOA	SSA	GWO	FDO	HHO	AEFA	PRO	WHO
F18	Min	1.9528E+03	2.1163E+03	2.1279E+03	2.8990E+03	2.4970E+03	3.3273E+03	5.7535E+09	2.0925E+03	2.4102E+03	1.9254E+03	**1.8204E+03**
	Max	4.3593E+04	2.7244E+04	4.7973E+04	4.9226E+04	5.3650E+04	5.4800E+04	9.5036E+09	3.8740E+04	1.9468E+04	3.8928E+04	**2.1465E+03**
	Avg	1.0734E+04	9.1500E+03	1.6258E+04	1.7733E+04	1.8817E+04	3.1931E+04	7.0988E+09	1.4672E+04	9.3202E+03	1.1095E+04	**1.8918E+03**
	Std	1.1036E+04	7.1402E+03	1.0061E+04	1.1207E+04	1.3679E+04	1.4487E+04	1.0187E+09	1.2135E+04	3.9428E+03	1.1737E+04	**7.3073E+01**
	Rank	4	2	7	8	9	10	11	6	3	5	1
F19	Min	1.9241E+03	1.9062E+03	1.9055E+03	2.2276E+03	1.9213E+03	1.9205E+03	2.1004E+07	1.9650E+03	4.3153E+03	1.9167E+03	**1.9012E+03**
	Max	8.5734E+03	2.4235E+04	1.9243E+03	1.5231E+05	8.9944E+03	1.6018E+04	6.5838E+09	2.8585E+04	1.4094E+05	2.9686E+04	**2.0217E+03**
	Avg	4.0595E+03	5.4847E+03	1.9142E+03	2.5420E+04	2.8197E+03	7.5592E+03	3.2867E+09	8.9542E+03	2.6483E+04	3.7764E+03	**1.9140E+03**
	Std	2.0294E+03	4.5303E+03	5.3598E+00	3.0016E+04	1.5980E+03	6.1398E+03	2.4058E+09	7.9662E+03	2.7832E+04	5.5398E+03	**2.6775E+01**
	Rank	5	6	2	9	3	7	11	8	10	4	1
F20	Min	2.0053E+03	**2.0011E+03**	2.0218E+03	2.0305E+03	2.0312E+03	2.0095E+03	2.2269E+03	2.0317E+03	2.0331E+03	2.0553E+03	2.0050E+03
	Max	2.3442E+03	**2.1205E+03**	2.2763E+03	2.3051E+03	2.1963E+03	2.1637E+03	2.3303E+03	2.2449E+03	2.3631E+03	2.4815E+03	2.1454E+03
	Avg	2.0838E+03	**2.0218E+03**	2.0868E+03	2.1674E+03	2.1047E+03	2.0486E+03	2.2376E+03	2.1263E+03	2.1642E+03	2.2097E+03	2.0307E+03
	Std	7.9273E+01	**2.2325E+01**	7.2957E+01	6.3453E+01	5.0779E+01	3.9872E+01	2.0487E+01	6.0591E+01	7.1718E+01	9.3169E+01	2.4712E+01
	Rank	4	1	5	9	6	3	11	7	8	10	2
Friedman mean rank		4.09	3.91	3.45	7.36	5.36	4.91	10.00	6.73	7.00	6.27	**1.10**
Rank		4	3	2	10	6	5	11	8	9	7	**1**

The bold numbers in a table are the best result that shown in each row of the table

that the proposed algorithm has the best performance in solving hybrid functions and has better performance in almost all functions than other compared algorithms. It is noteworthy that the results of the proposed algorithm significantly distanced from the results of other algorithms.

The results of the third group of CEC2017 functions shown in Table 7. The results show that the proposed algorithm in the Composition functions presents competitive results.

Figure 10 shows a complete comparison of the Friedman average rating of all algorithms on the CEC2017 test functions. As it is clear from the figure, the proposed algorithm has ranked best in all three groups of CEC2017 functions. This figure shows that the proposed algorithm can solve complex problems and can replace previous algorithms.

4.5 The performance of the wild horse optimizer on the CEC2019

In this section, we evaluate the performance of the proposed algorithm on the new test functions [16, 34]. We used the CEC2019 test functions. We used 30 search agents, 500 iterations and a maximum of 15,000 number evaluation function (NFE) for all algorithms. The results of this test shown in Table 8. The results show that the proposed algorithm performs well in solving new problems. However, some algorithms have very poor performance in solving new problems.

We used Friedman mean ranking for statistical analysis of algorithms. The results showed that the proposed algorithm is ranked first in solving CEC2019 problems. Figure 11 shows this ranking comparison.

4.6 The performance of the proposed algorithm on real problems

In this section, we evaluate the efficiency of the proposed algorithm in solving real problems in various scientific fields. The real problems are in fields such as industrial chemical processes, process design and composition, and mechanical engineering problems. Details of these problems given in Table 9. To observe the mathematical formulas of these problems, refer to Study [35]. Each method tested 30 times with 1000 iterations and a maximum of 60,000 number function evaluation (NFE).

4.6.1 Heat exchanger network design (case 2)

The first case is a heat exchange network design problem [36]. In this case, three non-linear equality constraints and six linear equality constraints with a non-linear objective function are involved in the problem. Moreover, seven additional linear inequality constraints included due to bounds on the temperatures. The results of this experiment shown in Table 10. As can be seen from the results of Table 10, the proposed algorithm has obtained the best optimal values in terms of statistics (minimum, maximum, mean, standard deviation and P value). The standard deviation and p value show that the proposed algorithm did not work to solve this problem based on chance, and the results are stable. The PSO algorithm ranks second, and other algorithms cannot solve this problem.

4.6.2 Process synthesis and design problem

The second problem is in the field of process composition and design [37]. These problems arise from the field of chemical engineering and with continuous and discrete variables, show non-convex optimization problems. The test results for this problem shown in Table 11. The results show that the proposed algorithm performed better than other algorithms and was able to obtain better optimal values. Statistical parameters prove this.

4.6.3 Two-reactor problem

The main purpose of this problem is to select one of the two reactors to optimize production costs. Details of this problem given in Study [38]. The test results of the proposed algorithm on this problem shown in Table 12. The results show that this problem, although it has a simple structure, cannot be solved with optimization algorithms. However, the proposed algorithm was able to discover the least optimal value. The poor and rich optimizer (PRO) algorithm was also able to take the second rank. Other algorithms have shown very poor performance.

4.6.4 Process synthesis problem

This problem provides seven degrees of freedom due to non-linearity in all real variables and binary variables [35]. Table 13 shows the results of this experiment. As can be seen from the results, the proposed algorithm and the Harris hawk optimizer (HHO) have the best performance compared to other algorithms. However, the proposed algorithm obtained a better optimal value than the HHO algorithm. The GA algorithm is not able to solve this problem. GSA and AEFA algorithms have performed poorly in solving this problem.

Table 7 Results for the CEC-2017 composition functions test functions with 10 dimension and 60,000 NFE

Function		PSO	GA	MVO	WOA	SSA	GWO	FDO	HHO	AEFA	PRO	WHO
F21	Min	2.2000E+03	2.2018E+03	2.2000E+03	2.2045E+03	2.2000E+03	2.2008E+03	2.4555E+03	2.2013E+03	2.2978E+03	2.2172E+03	**2.2000E+03**
	Max	2.3587E+03	2.3355E+03	2.3389E+03	2.4068E+03	2.3287E+03	2.3322E+03	2.6321E+03	2.3782E+03	2.3107E+03	2.4119E+03	**2.3149E+03**
	Avg	2.2991E+03	2.2955E+03	2.2839E+03	2.3246E+03	2.2600E+03	2.2936E+03	2.5279E+03	2.3104E+03	2.3062E+03	2.3620E+03	**2.2491E+03**
	Std	6.1421E+01	4.4121E+01	5.5832E+01	5.1930E+01	5.9450E+01	4.0762E+01	3.8410E+01	6.6374E+01	2.3946E+00	4.9953E+01	**5.4778E+01**
	Rank	6	5	3	9	2	4	11	8	7	10	1
F22	Min	2.2000E+03	2.2382E+03	**2.2000E+03**	2.2480E+03	2.3012E+03	2.3009E+03	3.4892E+03	2.2385E+03	2.3000E+03	2.4575E+03	2.2348E+03
	Max	3.1794E+03	2.3154E+03	**2.3068E+03**	3.5423E+03	2.3101E+03	2.3268E+03	4.5070E+03	2.3329E+03	2.3000E+03	4.4167E+03	2.3031E+03
	Avg	2.3222E+03	2.3050E+03	**2.2980E+03**	2.3530E+03	2.3037E+03	2.3063E+03	4.2107E+03	2.3122E+03	2.3000E+03	2.9558E+03	2.2987E+03
	Std	1.6429E+02	1.3144E+01	**2.3105E+01**	2.2526E+02	1.9900E+00	5.5988E+00	2.6199E+02	1.5194E+01	2.3885E−13	3.9690E+02	1.2085E+01
	Rank	8	5	1	9	4	6	11	7	3	10	2
F23	Min	2.6366E+03	**2.3001E+03**	2.6062E+03	2.6154E+03	2.6077E+03	2.6067E+03	2.9403E+03	2.6127E+03	2.6029E+03	2.6376E+03	2.6048E+03
	Max	2.7525E+03	2.6513E+03	2.6297E+03	2.6898E+03	2.6452E+03	2.6339E+03	3.4607E+03	2.7176E+03	**2.6224E+03**	2.7620E+03	2.6346E+03
	Avg	2.6724E+03	2.6200E+03	2.6183E+03	2.6497E+03	2.6222E+03	2.6171E+03	3.3329E+03	2.6592E+03	**2.6113E+03**	2.6865E+03	2.6134E+03
	Std	2.8527E+01	6.1239E+01	6.4643E+00	1.7806E+01	1.0266E+01	7.9052E+00	1.6606E+02	2.4988E+01	**5.2540E+00**	3.3814E+01	6.7487E+00
	Rank	9	5	4	7	6	3	11	8	1	10	2
F24	Min	2.5000E+03	2.5001E+03	2.5001E+03	2.5060E+03	2.7307E+03	2.7001E+03	3.1137E+03	2.5011E+03	2.5000E+03	2.7719E+03	**2.5000E+03**
	Max	2.8792E+03	**2.7912E+03**	2.7597E+03	2.8340E+03	2.7691E+03	2.7652E+03	3.2587E+03	2.9153E+03	2.7409E+03	2.9705E+03	2.7560E+03
	Avg	2.7636E+03	**2.6876E+03**	2.7213E+03	2.7409E+03	2.7465E+03	2.7435E+03	3.2068E+03	2.7919E+03	2.7162E+03	2.8378E+03	2.7342E+03
	Std	1.0998E+02	**1.0950E+02**	7.5290E+01	1.0283E+02	8.6722E+00	1.3097E+01	3.7169E+01	8.5570E+01	5.8975E+01	4.2515E+01	4.4490E+01
	Rank	8	1	3	5	7	6	11	9	2	10	4
F25	Min	**2.8977E+03**	2.8979E+03	2.8978E+03	2.9006E+03	2.8978E+03	2.8981E+03	3.3768E+03	2.7261E+03	2.9440E+03	3.0252E+03	2.8982E+03
	Max	**2.9459E+03**	2.9543E+03	2.9461E+03	3.0294E+03	2.9694E+03	2.9495E+03	4.2772E+03	3.0246E+03	2.9520E+03	4.6806E+03	2.9526E+03
	Avg	**2.9180E+03**	2.9338E+03	2.9200E+03	2.9462E+03	2.9276E+03	2.9377E+03	3.7836E+03	2.9234E+03	2.9491E+03	3.7017E+03	2.9242E+03
	Std	**2.3824E+01**	2.2910E+01	2.3453E+01	2.7072E+01	2.4601E+01	1.5851E+01	2.4731E+02	4.8503E+01	2.1981E+00	5.1478E+02	2.4168E+01
	Rank	1	6	2	8	5	7	11	3	9	10	4
F26	Min	2.6000E+03	2.6006E+03	2.9000E+03	2.9020E+03	2.8000E+03	2.6010E+03	4.7781E+03	2.8168E+03	**2.8000E+03**	3.1994E+03	2.8000E+03
	Max	3.9118E+03	3.7859E+03	3.9989E+03	4.3323E+03	3.0344E+03	3.9285E+03	5.1980E+03	4.4903E+03	**2.9000E+03**	5.2393E+03	3.8095E+03
	Avg	3.1141E+03	3.0486E+03	3.0047E+03	3.5179E+03	2.9132E+03	3.0823E+03	5.0851E+03	3.2883E+03	**2.8600E+03**	4.3155E+03	2.9668E+03
	Std	2.9980E+02	2.0495E+02	3.0889E+02	5.7785E+02	4.2062E+01	3.6691E+02	1.2507E+02	4.5924E+02	**4.9827E+01**	6.5526E+02	1.7158E+02
	Rank	7	5	4	9	2	6	11	8	1	10	3
F27	Min	3.1027E+03	3.0921E+03	3.0887E+03	3.0941E+03	**3.0888E+03**	3.0898E+03	3.2000E+03	3.1008E+03	3.0968E+03	3.0987E+03	3.0883E+03
	Max	3.2958E+03	3.1474E+03	3.1620E+03	3.2068E+03	**3.0975E+03**	3.1811E+03	4.1571E+03	3.2521E+03	3.1946E+03	3.2671E+03	3.1085E+03
	Avg	3.1615E+03	3.1126E+03	3.0935E+03	3.1212E+03	**3.0920E+03**	3.1000E+03	3.8248E+03	3.1510E+03	3.1135E+03	3.1545E+03	3.0949E+03
	Std	4.7787E+01	1.6048E+01	1.3079E+01	3.4780E+01	**2.8458E+00**	1.9562E+01	3.4940E+02	4.2197E+01	2.2009E+01	3.6090E+01	4.2854E+00
	Rank	10	5	2	7	1	4	11	8	6	9	3

Table 7 (continued)

Function		PSO	GA	MVO	WOA	SSA	GWO	FDO	HHO	AEFA	PRO	WHO
F28	Min	3.1000E+03	3.1000E+03	3.1001E+03	3.1116E+03	**3.1000E+03**	3.1627E+03	3.2996E+03	3.1653E+03	3.2137E+03	3.2880E+03	3.1000E+03
	Max	3.4465E+03	3.4465E+03	3.4118E+03	3.7494E+03	**3.7318E+03**	3.5029E+03	4.2060E+03	3.7494E+03	3.4585E+03	3.9109E+03	3.4120E+03
	Avg	3.3640E+03	3.2578E+03	3.3338E+03	3.3574E+03	**3.2359E+03**	3.3503E+03	3.9733E+03	3.3895E+03	3.3892E+03	3.3857E+03	3.2722E+03
	Std	9.5907E+01	1.3968E+02	1.2192E+02	1.4888E+02	**1.4939E+02**	1.0737E+02	2.7036E+02	1.2884E+02	4.5644E+01	1.5474E+02	1.4039E+02
	Rank	7	2	4	6	1	5	11	9	8	10	3
F29	Min	3.1469E+03	3.1580E+03	3.1360E+03	3.1835E+03	3.1317E+03	3.1443E+03	4.4422E+03	3.2069E+03	3.1997E+03	3.1918E+03	**3.1349E+03**
	Max	3.3397E+03	3.3293E+03	3.3524E+03	3.4850E+03	3.3402E+03	3.3306E+03	8.2249E+03	3.4956E+03	3.6283E+03	3.5163E+03	**3.2082E+03**
	Avg	3.2250E+03	3.2027E+03	3.2182E+03	3.3146E+03	3.2099E+03	3.1939E+03	5.7344E+03	3.3087E+03	3.3334E+03	3.3244E+03	**3.1581E+03**
	Std	4.8234E+01	3.6859E+01	6.9631E+01	8.5355E+01	6.1739E+01	5.0433E+01	8.8061E+02	7.0303E+01	1.0385E+02	9.5773E+01	**1.5695E+01**
	Rank	6	3	5	8	4	2	11	7	10	9	1
F30	Min	4.0214E+03	4.9304E+04	5.9436E+03	8.2466E+03	1.4597E+04	7.4553E+03	8.6196E+07	7.4431E+03	2.5776E+05	6.1378E+03	**3.4612E+03**
	Max	1.2533E+06	5.0784E+06	1.3982E+06	3.9598E+06	1.8755E+06	2.9768E+06	3.1601E+08	4.5390E+06	8.7012E+06	7.3381E+06	**8.2058E+05**
	Avg	2.7642E+05	5.5446E+05	2.7247E+05	5.8014E+05	3.6497E+05	6.3093E+05	2.2912E+08	9.5621E+05	2.3217E+06	1.5284E+06	**1.4050E+05**
	Std	4.3354E+05	1.0435E+06	4.6350E+05	7.9909E+05	5.9710E+05	8.9413E+05	5.9599E+07	1.0147E+06	2.0381E+06	1.8281E+06	**3.0934E+05**
	Rank	3	5	2	6	4	7	11	8	10	9	1
Friedman mean rank		5.91	3.82	2.73	6.73	3.27	4.55	10.00	6.82	5.18	8.82	**2.18**
Rank		7	4	2	8	3	5	11	9	6	10	**1**

The bold numbers in a table are the best result that shown in each row of the table

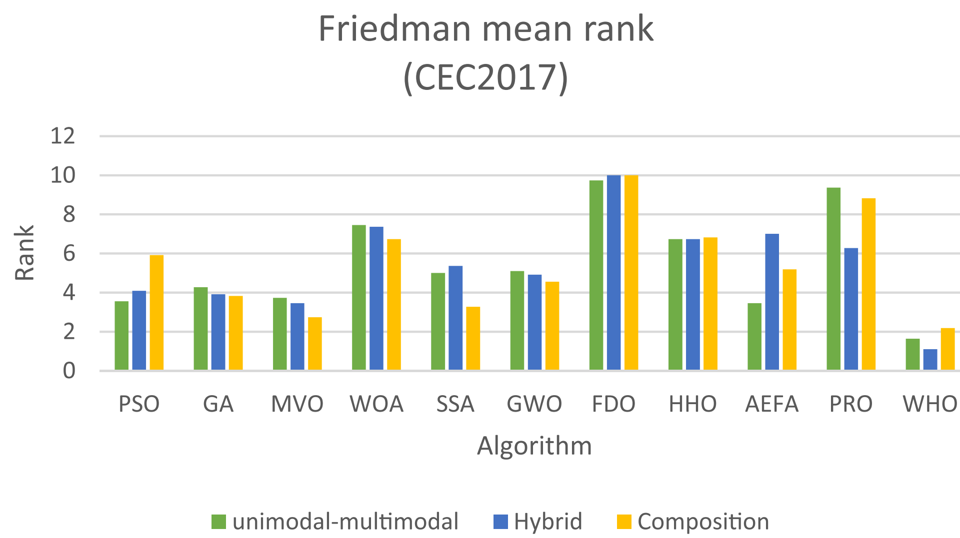


Fig. 10 Friedman mean ranking of CEC2017 functions

4.6.5 Tension/compression spring design

The main purpose of this problem is to optimize the weight of the tension or compression spring [39]. This problem consists of four constraints, and three variables used to calculate the weight: wire diameter, average coil diameter, and the number of active coils. The results of this experiment shown in Table 14. The results showed that the PRO algorithm had the best performance in solving this problem. The results of the proposed algorithm are close to the results of the PRO algorithm, and it was able to allocate the second rank.

4.6.6 Three-bar truss design problem

This optimization problem taken from civil engineering, which has a problematic, constrained space [40]. The main objective of this problem is to minimize the weight of the bar structures. The constraints of this problem based on the stress constraints of each bar. The resultant problem is a non-linear objective function with three non-linear constraints. The results shown in Table 15. The results showed that the proposed algorithm was able to discover the best optimal value for this problem. The PRO algorithm cannot solve this problem.

4.6.7 Step-cone pulley problem

The main objective of this problem is to minimize the weight of four step-cone pulleys using five variables in which four variables are the diameters of each step of the pulley, and the last one is the width of the pulley [41]. This problem contains 11 non-linear constraints to assure that the transmit power must be at 0.75 hp. The results of this experiment shown in Table 16. As can be seen from the results, in terms of the average, the PSO algorithm performed better than other algorithms. However, the least optimal value discovered by the proposed algorithm. It is noteworthy that the proposed algorithm is ranked second and has a better performance than other compared algorithms.

4.6.8 Rolling element bearing

In this problem, the difference between the minimum and maximum force generated by the robot gripper used as an objective function [42]. This problem contains seven design variables and six non-linear design constraints associated with the robot. The test results shown in Table 17. As can be seen from Table 17, the results of the algorithms are close to each other. However, the proposed algorithm was able to solve this problem more accurately. The PRO algorithm is not able to solve this problem.

Table 8 Results for the 2019 unimodal and multimodal benchmark functions

Function		PSO	GA	GSA	MVO	SSA	FDO	AEFA	HHO	GWO	PRO	WHO
F1	Min	3.6316e+06	1.2909e+09	9.9600e+11	6.4716e+08	2.5399e+08	3.6147e+10	7.0736e+11	7.1152e+04	1.1496e+07	5.7703e+04	**4.0179e+04**
	Max	1.7475e+09	7.0070e+10	6.8573e+12	6.0026e+09	4.8189e+10	3.5336e+11	5.7444e+12	1.1493e+05	1.6487e+09	5.8247e+05	**1.0433e+05**
	Avg	3.7481e+08	1.8005e+10	3.6523e+12	3.2466e+09	9.8966e+09	1.9700e+11	2.8575e+12	9.0519e+04	5.3500e+08	2.2000e+05	**5.4799e+04**
	Std	5.2101e+08	2.0443e+10	2.0735e+12	1.8839e+09	1.4662e+10	1.0567e+11	1.5564e+12	1.3772e+04	5.8485e+08	1.4263e+05	**2.0301e+04**
	Rank	4	8	11	6	7	9	10	2	5	3	**1**
F2	Min	1.7343E+01	1.7359E+01	5.7902E+03	1.7903E+01	1.7343E+01	5.3867E+02	8.5573E+03	1.7388E+01	1.7343E+01	1.7382E+01	**1.7343E+01**
	Max	1.7343E+01	7.4163E+01	2.7841E+04	2.4167E+01	1.7392E+01	2.9384E+03	2.0064E+04	1.7798E+01	1.7679E+01	1.8426E+01	**1.7343E+01**
	Avg	1.7343E+01	2.8498E+01	1.6143E+04	1.9775E+01	1.7346E+01	1.2600E+03	1.4268E+04	1.7481E+01	1.7355E+01	1.7755E+01	**1.7343E+01**
	Std	8.3709E−15	1.4505E+01	4.9858E+03	1.5118E+00	9.7000E−03	5.0223E+02	3.5437E+03	9.4200E−02	6.1100E−02	2.8480E−01	**7.0129E−15**
	Rank	2	8	11	7	3	9	10	5	4	6	**1**
F3	Min	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	**1.2702E+01**
	Max	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2704E+01	1.2705E+01	1.2705E+01	1.2703E+01	1.2702E+01	1.2707E+01	**1.2702E+01**
	Avg	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2703E+01	1.2703E+01	1.2703E+01	1.2702E+01	1.2702E+01	1.2703E+01	**1.2702E+01**
	Std	3.6134E−15	2.8453E−07	3.6134E−15	6.9400E−09	3.8383E−04	5.2674E−04	6.2782E−04	2.0012E−05	9.7975E−07	1.0000E−03	**3.6134E−15**
	Rank	1	3	1	2	6	8	7	5	4	9	**1**
F4	Min	**2.9849E+00**	5.4231E+01	4.9747E+01	7.8549E+00	1.8010E+02	2.2929E+03	7.9597E+00	9.6036E+02	1.9553E+01	5.1518E+03	1.2945E+01
	Max	**4.3778E+01**	2.5008E+02	7.5397E+02	5.6456E+01	1.4427E+02	8.5861E+03	3.3328E+02	6.3391E+03	1.0102E+03	3.3003E+04	8.5566E+01
	Avg	**1.9254E+01**	1.2905E+02	3.2007E+02	3.0091E+01	4.5237E+01	4.7371E+03	6.9048E+01	2.9775E+03	8.9310E+01	1.3463E+04	4.1667E+01
	Std	**9.5931E+00**	4.8556E+01	1.8010E+02	1.0484E+01	2.8066E+01	1.6869E+03	6.9642E+01	1.4929E+03	1.7519E+02	6.4678E+03	2.0143E+01
	Rank	**1**	7	8	2	4	10	5	9	6	11	3
F5	Min	1.0271E+00	1.0800E+00	1.0000E+00	1.0493E+00	1.0565E+00	2.1985E+00	**1.0000E+00**	2.0672E+00	1.0931E+00	3.6332E+00	1.0000E+00
	Max	1.3172E+00	1.5119E+00	1.2853E+00	1.4777E+00	1.7650E+00	3.3416E+00	**1.0246E+00**	4.3798E+00	1.8013E+00	7.9354E+00	1.3297E+00
	Avg	1.1412E+00	1.1972E+00	1.0596E+00	1.2804E+00	1.2452E+00	2.5252E+00	**1.0065E+00**	3.0792E+00	1.3684E+00	5.8830E+00	1.0761E+00
	Std	8.6000E−02	1.0100E−01	6.3500E−02	9.6000E−02	1.8100E−01	2.6310E−01	**8.5000E−03**	5.9360E−01	2.4550E−01	1.2290E+00	5.9800E−02
	Rank	4	5	2	7	6	9	**1**	10	8	11	3
F6	Min	5.4320E+00	3.5465E+00	1.0001E+00	6.3180E+00	1.9834E+00	9.5286E+00	**1.0000E+00**	8.1858E+00	8.3174E+00	8.3310E+00	4.2775E+00
	Max	1.0618E+01	7.0036E+00	1.0004E+00	1.1539E+01	9.9586E+00	1.4622E+01	**1.0000E+00**	1.1975E+01	1.2190E+01	1.2203E+01	7.7699E+00
	Avg	8.6324E+00	5.5461E+00	1.0002E+00	8.6024E+00	5.5108E+00	1.3080E+01	**1.0000E+00**	1.0499E+01	1.0969E+01	1.0601E+01	5.7176E+00
	Std	1.4167E+00	8.2840E−01	6.4518E−05	1.4295E+00	1.8155E+00	1.0924E+00	**1.2890E−06**	9.2490E−01	7.3060E−01	8.7810E−01	8.6930E−01
	Rank	7	4	2	6	3	11	**1**	8	10	9	5
F7	Min	−6.4755E+01	−1.5598E+02	1.0278E+02	−1.3689E+02	−2.8775E+01	6.0381E+02	2.3396E+01	1.4936E+02	−7.4650E+01	6.2703E+02	**−1.4828E+02**
	Max	3.8122E+02	4.8727E+02	5.9472E+02	6.4962E+02	7.3926E+02	1.6107E+03	6.3436E+02	1.0611E+03	9.6256E+02	1.5726E+03	**4.6500E+02**
	Avg	1.4332E+02	2.2106E+02	2.5189E+02	2.6614E+02	3.2398E+02	1.1451E+03	2.5078E+02	6.0227E+02	4.4960E+02	1.0436E+03	**1.0287E+02**
	Std	1.0307E+02	1.5239E+02	1.0860E+02	2.1001E+02	1.8238E+02	2.5241E+02	1.2445E+02	2.4814E+02	2.8871E+02	2.5135E+02	**1.3584E+02**
	Rank	2	3	5	6	7	11	4	9	8	10	**1**

Table 8 (continued)

Function		PSO	GA	GSA	MVO	SSA	FDO	AEFA	HHO	GWO	PRO	WHO
F8	Min	3.2864E+00	3.3511E+00	4.7733E+00	3.5296E+00	3.9768E+00	5.5140E+00	3.3300E+00	4.6175E+00	3.2557E+00	4.8443E+00	**2.9417E+00**
	Max	6.1854E+00	6.0462E+00	6.6863E+00	6.2702E+00	6.1784E+00	8.0925E+00	6.1525E+00	7.0294E+00	7.0385E+00	7.1783E+00	**5.8683E+00**
	Avg	4.9537E+00	4.9410E+00	5.6491E+00	5.2702E+00	5.2243E+00	6.5832E+00	5.1132E+00	6.0216E+00	5.0215E+00	6.2528E+00	**4.8852E+00**
	Std	7.5560E−01	7.4110E−01	5.0050E−01	8.2450E−01	6.1620E−01	6.3830E−01	6.3890E−01	5.4820E−01	9.9640E−01	5.2350E−01	**6.7260E−01**
	Rank	3	2	8	7	6	11	5	9	4	10	**1**
F9	Min	**2.3430E+00**	2.5288E+00	2.9813E+00	2.3704E+00	2.4273E+00	2.0434E+02	2.7281E+00	5.1157E+00	2.7351E+00	6.6163E+02	2.3427E+00
	Max	**2.3893E+00**	4.0652E+00	5.9368E+00	2.7501E+00	3.4095E+00	1.0507E+03	5.4750E+00	3.9306E+02	6.4618E+00	4.5913E+03	3.1306E+00
	Avg	**2.3628E+00**	3.0440E+00	4.1548E+00	2.4669E+00	2.7059E+00	6.5210E+02	3.8273E+00	1.1902E+02	4.2653E+00	2.8887E+03	2.5672E+00
	Std	**1.3400E−02**	3.2930E−01	8.4080E−01	8.2400E−02	2.4080E−01	2.4820E+02	6.3820E−01	9.6599E+01	9.8460E−01	1.0744E+03	1.7810E−01
	Rank	**1**	5	7	2	4	10	6	9	8	11	3
F10	Min	1.9644E−11	2.7150E+00	1.9958E+01	2.0014E+01	1.9999E+01	2.0415E+01	**8.6064E−13**	2.0092E+01	2.0095E+01	2.0200E+01	2.8160E+00
	Max	2.0535E+01	2.0071E+01	1.9999E+01	2.0280E+01	2.0428E+01	2.0986E+01	**2.0000E+01**	2.0566E+01	2.0650E+01	2.0697E+01	2.0041E+01
	Avg	1.9537E+01	1.9450E+01	1.9985E+01	2.0108E+01	2.0042E+01	2.0727E+01	**1.7995E+01**	2.0338E+01	2.0466E+01	2.0457E+01	1.9439E+01
	Std	3.6939E+00	3.1609E+00	1.0500E−02	6.2400E−02	9.9500E−02	1.4190E−01	**6.1008E+00**	1.2300E−01	1.1780E−01	1.1690E−01	3.1395E+00
	Rank	4	3	5	7	6	11	1	8	10	9	2
Friedman mean rank		2.64	4.36	5.45	4.72	4.73	9.18	4.18	6.73	6.09	8.09	**1.91**
Rank		2	4	7	5	6	11	3	9	8	10	**1**

The bold numbers in a table are the best result that shown in each row of the table

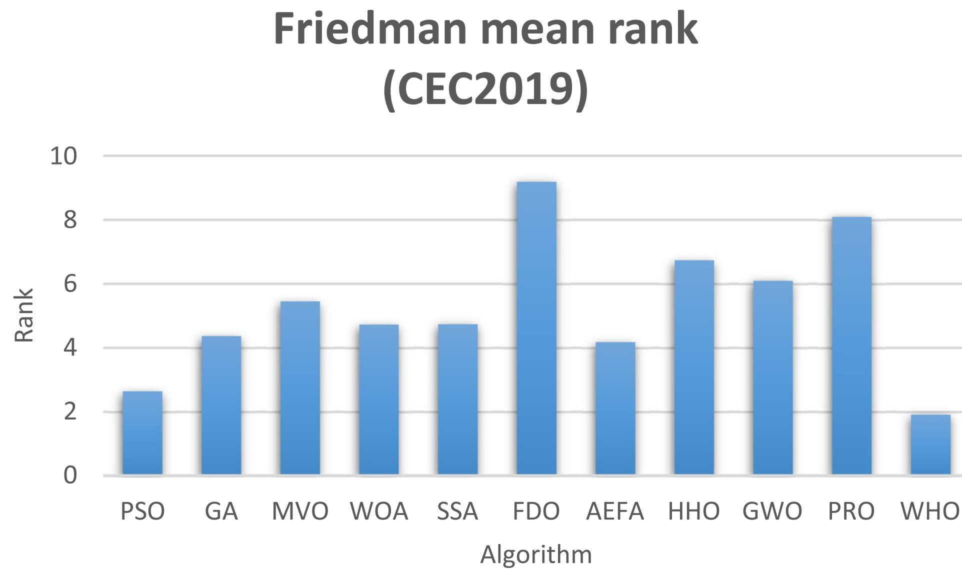


Fig. 11 Friedman mean ranking on CEC2019 test functions

Table 9 Details of the ten real-world constrained optimization problem. D is the total number of decision variables of the problem, g is the number of inequality constraints, and h is the number of equality constraints, $F(x)$ is best known feasible objective function value

Prob	Name	D	g	h	$F(x)$
Industrial chemical processes					
1	Heat exchanger network design (case 2)	11	0	9	7.0490369540E+03
Process synthesis and design problems					
2	Process synthesis and design problem	3	1	1	2.5576545740E+00
3	Two-reactor problem	7	4	4	9.9238463653E+01
4	Process synthesis problem	7	9	0	2.9248305537E+00
Mechanical engineering problem					
5	Tension/compression spring design (case 1)	3	3	0	1.2665232788E−02
6	Three-bar truss design problem	2	3	0	2.6389584338E+02
7	Step-cone pulley problem	5	8	3	1.6069868725E+01
8	Rolling element bearing	10	9	0	1.4614135715E+04
9	Gas transmission compressor design (GTCD)	4	1	0	2.9648954173E+06
10	Himmelblau's function	5	6	0	− 3.0665538672E+04

Table 10 Results for heat exchanger network design (case 2)

Algorithm	Optimum cost				
	Min	Max	Std	Avg	p value
WHO	**7.0490e+03**	**7.0490e+03**	**2.0063e−04**	**7.0490e+03**	**N/A**
PSO	7.0491e+03	7.0569e+03	1.6794	7.0500e+03	1.9292e−11
GA	4.4982e+22	3.6019e+23	6.6458e+22	1.9241e+23	1.9292e−11
AEFA	5.6937e+26	3.7784e+27	8.4895e+26	2.6768e+27	1.9292e−11
FA	6.4635e+17	3.4740e+18	7.7177e+17	2.0704e+18	1.9292e−11
GSA	8.7539e+26	5.9206e+27	1.2413e+27	3.0422e+27	1.9292e−11
HHO	1.2826e+22	7.2822e+25	2.8207e+25	2.2968e+25	1.9292e−11
MVO	8.2571e+19	4.3520e+23	7.9501e+22	2.6570e+22	1.9292e−11
WOA	3.0755e+22	8.1185e+24	1.5564e+24	1.7597e+24	1.9292e−11
SSA	1.7907e+16	1.0938e+24	2.6277e+23	1.3063e+23	1.9292e−11
GWO	1.0876e+21	3.8313e+22	9.2541e+21	1.2213e+22	1.9292e−11
PRO	2.4165e+24	7.4633e+26	1.8967e+26	3.4613e+26	1.9292e−11

The bold numbers in a table are the best result that shown in each row of the table

Table 11 Results for process synthesis and design problem

Algorithm	Optimum cost				
	Min	Max	Std	Avg	*p* value
WHO	**2.5578E+00**	**2.6689E+00**	**3.2700E-02**	**2.5878E+00**	–
PSO	2.5637E+00	2.9261E+00	1.0860E−01	2.6914E+00	3.8349E−06
GA	2.5608E+00	3.2588E+03	8.2950E+02	2.5061E+02	3.5708E−06
AEFA	2.6194E+00	2.9261E+00	7.0300E−02	2.9062E+00	5.3091E−11
FA	2.5587E+00	1.3525E+01	2.2342E+00	3.5610E+00	1.1023E−08
GSA	2.5614E+00	2.7148E+00	4.1000E−02	2.6118E+00	5.6000E−03
HHO	2.5662E+00	3.7558E+00	3.1970E−01	2.8792E+00	9.2559E−09
MVO	2.8219E+00	3.2982E+03	9.9231E+02	6.0221E+02	3.0199E−11
WOA	2.5734E+00	3.9262E+00	3.7680E−01	2.9094E+00	3.4971E−09
SSA	2.5588E+00	3.3885E+00	2.0270E−01	2.7207E+00	9.7917E−05
GWO	4.2381E+00	2.9254E+04	5.9560E+03	3.0915E+03	3.0199E−11
PRO	2.5949E+00	3.8802E+00	4.7380E−01	2.9870E+00	5.4617E−09

The bold numbers in a table are the best result that shown in each row of the table

4.6.9 Gas transmission compressor design (GTCD)

This problem is formulated as follows [43].

Minimize：

$$f(\bar{x}) = 8.16 \times 10^5 x_1^{1/2} x_2 x_3^{-2/3} x_4^{-1/2} + 3.69 \times 10^4 x_3 + 7.72 \times 10^8 x_1^{-1} x_2^{0.219} - 765.43 \times 10^6 x_1^{-1}$$

subject to：

$$x_4 x_2^{-2} + x_2^{-2} - 1 \le 0,$$

with bound 1s：

$$20 \le x_1 \le 50,$$
$$1 \le x_2 \le 10,$$
$$20 \le x_3 \le 50,$$
$$0.1 \le x_4 \le 60.$$

The results of this experiment shown in Table 18. The results showed that the proposed algorithm had the best performance in solving this problem and was able to discover the best optimal value. The PRO algorithm has been unable to solve this problem.

4.6.10 Himmelblau's function

Himmelblau proposes this problem which is used as a standard benchmark problem to analyze the non-linear constrained optimization algorithms [44]. This problem contains six non-linear constraints and five variables. The results of this experiment shown in Table 19. The results

Table 12 Results for two-reactor problem

Algorithm	Optimum cost			
	Min	Max	Std	Avg
WHO	**9.9265E+01**	**1.0000E+15**	**5.0742E+14**	**4.6667E+14**
PSO	1.0000E+15	1.0000E+15	1.2149E+01	1.0000E+15
GA	1.0000E+15	1.0000E+15	1.9268E+04	1.0000E+15
AEFA	1.6341E+14	1.0000E+15	2.3082E+14	8.9422E+14
FA	1.0660E+07	1.0000E+15	1.8257E+14	9.6667E+14
GSA	1.0000E+15	1.0000E+15	3.8456E+00	1.0000E+15
HHO	5.6717E+07	1.0000E+15	3.4565E+14	8.6670E+14
MVO	1.9737E+08	1.0000E+15	4.6609E+14	7.0000E+14
WOA	2.1256E+06	1.0000E+15	3.9418E+14	8.0674E+14
SSA	2.6001E+02	1.0000E+15	1.8257E+14	9.6667E+14
GWO	4.7907E+08	1.0000E+15	4.8926E+14	6.3399E+14
PRO	1.0066E+02	1.8372E+16	3.1935E+15	1.5140E+15

The bold numbers in a table are the best result that shown in each row of the table

Table 13 Results for process synthesis problem

Algorithm	Optimum cost				
	Min	Max	Std	Avg	*p* value
WHO	**2.924830553663483**	**4.206505499743916**	**0.457682489025943**	**3.157838122984132**	N/A
PSO	2.924830673734528	4.767500610053721	0.722844225068448	3.608737061618407	2.0000E−03
GA	–	–	–	–	
AEFA	13.306852819440055	6.891369399323785e+22	1.711577820553067e+22	6.306687874818297e+21	2.6859E−11
FA	2.925027883032334	4.900611369373891	0.689215574327478	3.734594407343399	1.2117E−06
GSA	9.914767571690899e+16	2.117849949159403e+22	6.303397800546704e+21	4.720786116567751e+21	2.7047E−11
HHO	2.924911684704210	4.691090767395534	0.356034464499842	**3.080887906833858**	2.6700E−02
MVO	2.929556053076952	4.928228360588580	0.663579190782002	3.638060537653593	3.3806E−06
WOA	2.934740643734108	10.824353430053057	2.194063404067135	5.480104789362980	3.8352E−09
SSA	2.924831711891477	4.899677686295459	0.670273221145556	3.729343546892578	7.8814E−06
GWO	2.924913897646869	5.507392933392770	0.840618022369924	3.828759444847278	1.0000E−03
PRO	3.082150687123927	5.253448191411385	0.471595491512416	4.600216615832678	1.2574E−10

The bold numbers in a table are the best result that shown in each row of the table

Table 14 Results for tension/compression spring design (case 1)

Algorithm	Optimum cost				
	Min	Max	Std	Avg	*p* value
WHO	0.012665236818810	0.012824673596214	3.820959940393885e−05	0.012700124939337	6.0621e−11
PSO	0.012669198362236	0.014489992405201	5.004907572131879e−04	0.013260742035549	2.0328e−09
GA	0.012788721689533	0.018332902912960	0.001607219194489	0.015449417442230	5.1838e−07
AEFA	0.012683490907962	0.013904676465493	2.877738620059324e−04	0.012924229038656	0.0064
FA	0.012666255993050	0.013428153934211	1.518452383136235e−04	0.012766102878197	0.0064
GSA	0.013601629996108	0.021723571363358	0.001798141523560	0.016606397924405	3.0180e−11
HHO	0.012674749431788	0.017773158747079	0.001284233426480	0.014042838434392	3.4722e−10
MVO	0.012813487715808	0.018095031858868	0.001472488137202	0.017117505869731	3.3363e−11
WOA	0.012672314464682	0.017773195197909	0.001187560003287	0.013571130817365	3.3505e−08
SSA	0.012723064443054	0.017726329576613	0.001043440251874	0.013268849047859	2.1532e−10
GWO	0.012675187367657	0.012796561736371	2.127396335428537e−05	0.012718855033841	8.5627e−04
PRO	**0.012665232788319**	**0.012665452963227**	**4.033298012484218e−08**	**0.012665242163550**	**–**

The bold numbers in a table are the best result that shown in each row of the table

show that the proposed algorithm has worked well in solving this problem and has been able to discover the best optimal value. Statistical parameters show that the results of the proposed algorithm are stable and not based on chance.

5 Conclusion

This paper proposed a nature-inspired optimization algorithm called the wild horse algorithm and mimicked the behaviour of wild horses in nature. The special inspiration

Table 15 Results for three-bar truss design problem

Algorithm	Optimum cost				
	Min	Max	Std	Avg	*p* value
WHO	**2.638958433764640e+02**	**2.638958433764640e+02**	**1.271057486462604e-13**	**2.638958433764639e+02**	**–**
PSO	2.638958433827166e+02	2.638960409745313e+02	5.391716111901965e−05	2.638959010895359e+02	5.1436e−12
GA	2.638958919373082e+02	2.639970875475242e+02	0.025205557688814	2.639095296975827e+02	5.1436e−12
AEFA	2.651001279647231e+02	2.809534461900492e+02	4.055862568613710	2.718733092380326e+02	5.1436e−12
FA	2.638958477145308e+02	2.638989975835582e+02	8.845534498369423e−04	2.638964634152982e+02	5.1436e−12
GSA	2.638968857659676e+02	2.641972851297597e+02	0.094894105625134	2.640059193538127e+02	5.1436e−12
HHO	2.638959528569774e+02	2.640672685182423e+02	0.046762128703369	2.639419743128567e+02	5.1436e−12
MVO	2.638958747018914e+02	2.639000377232526e+02	9.860139749906427e−04	2.638967256362468e+02	5.1436e−12
WOA	2.638959383525027e+02	2.656916186134478e+02	0.502907430613907	2.643105859276705e+02	5.1436e−12
SSA	2.638958435095750e+02	2.638998220362238e+02	7.267874787295818e−04	2.638962415756725e+02	5.1436e−12
GWO	2.638959818300091e+02	2.639028435626118e+02	0.001437171411278	2.638975822283781e+02	5.1436e−12
PRO	− 1.600200140746955e+03	− 1.102920485233276e+03	− 1.283881889035918e+03	1.196996140045274e+02	

The bold numbers in a table are the best result that shown in each row of the table

Table 16 Results for step-cone pulley problem

Algorithm	Optimum cost				
	Min	Max	Std	Avg	*p* value
WHO	**1.6090E+01**	1.7364E+01	4.6120E-01	1.6776E+01	–
PSO	1.6133E+01	**1.7136E+01**	**3.2730E-01**	**1.6706E+01**	1.6680E−01
GA	1.6353E+01	9.2889E+07	2.6916E+07	1.9800E+07	3.0900E−06
AEFA	1.6955E+01	1.7821E+01	2.8510E−01	1.7429E+01	3.0057E−07
FA	5.0238E+02	7.8113E+03	1.9691E+03	3.5119E+03	3.0123E−11
GSA	1.6186E+01	4.9336E+11	1.3894E+11	1.1315E+11	3.7996E−07
HHO	1.6534E+01	1.8926E+08	3.4553E+07	6.3085E+06	1.0263E−06
MVO	1.4997E+04	8.1181E+05	1.5458E+05	1.6969E+05	3.0123E−11
WOA	1.6913E+01	4.7707E+10	9.0540E+09	2.1132E+09	4.1904E−10
SSA	1.6219E+01	1.8090E+01	5.0310E−01	1.7361E+01	4.2144E−04
GWO	5.7176E+05	1.0435E+07	2.5057E+06	3.9957E+06	3.0123E−11
PRO	1.6293E+01	1.1314E+13	2.3685E+12	1.0038E+12	3.5056E−05

The bold numbers in a table are the best result that shown in each row of the table

of this algorithm is the special and unique mating behaviour of wild horses. So that members of a family cannot mate with each other, and when they reach puberty, they should leave the group and join other groups and find their mate. Another important behaviour is grazing the group horses around the stallion or the group leader. For the effectiveness of the proposed algorithm, an adaptive parameter was used, which helped to solve complex problems better. To demonstrate the efficiency of the proposed algorithm, unimodal, multimodal, hybrid, hybrid, and new

Table 17 Results for rolling element bearing

Algorithm	Optimum cost				
	Min	Max	Std	Avg	*p* value
WHO	**1.461413571502580e + 04**	**1.461413571502582e+04**	**8.775726031253865e−12**	**1.461413571502582e+04**	–
PSO	1.461413571502581e+04	1.461413571502614e+04	7.324562998706849e−11	1.461413571502587e+04	1.1552e−11
GA	1.461477420743422e+04	1.463995782639870e+04	6.672100573998030	1.462440275893069e+04	3.1578e−12
AEFA	1.468411477275932e+04	1.495567890295754e+04	47.885030015570685	1.470284090916723e+04	3.1578e−12
FA	1.470906831178014e+04	1.526475354613637e+04	1.447327692792185e+02	1.496596066392380e+04	3.1578e−12
GSA	2.003358626934052e+04	3.193377188720165e+04	2.953068325682148e+03	2.623031617797562e+04	3.1578e−12
HHO	1.461413594004388e+04	2.028380024811414e+04	1.027113227228814e+03	1.486011191277977e+04	3.1554e−12
MVO	1.462798024919465e+04	1.473891805135771e+04	29.811984095120092	1.467355933888599e+04	3.1578e−12
WOA	1.461536420741418e+04	1.553906688169088e+04	3.092380158593184e+02	1.491744542285628e+04	3.1578e−12
SSA	1.464038660842310e+04	1.549674891802942e+04	3.109179956704083e+02	1.494584705287445e+04	3.1578e−12
GWO	1.462050034545684e+04	1.468842154047419e+04	17.966371315913534	1.464717066780431e+04	3.1578e−12
PRO	1.936995400085942e+04	− Inf	Inf	Inf	Inf

The bold numbers in a table are the best result that shown in each row of the table

Table 18 Results for gas transmission compressor design (GTCD)

Algorithm	Optimum cost				
	Min	Max	Std	Avg	*p* value
WHO	**2.964895415859130e+06**	**2.964895415859133e+06**	**1.124124692893435e−09**	**2.964895415859132e+06**	–
PSO	2.964895507894898e+06	2.966295851848677e+06	2.957278241168343e+02	2.965029875097693e+06	2.2333e−11
GA	3.260070708065281e+06	7.549584007575896e+06	1.045006396884415e+06	5.517711951461810e+06	2.2333e−11
AEFA	3.322646459862843e+06	4.026899365263686e+06	1.802386695761881e+05	3.538910764517806e+06	2.2333e−11
FA	2.966345889928184e+06	3.199751698170416e+06	6.092533954792276e+04	3.052479798682354e+06	2.2333e−11
GSA	3.314277721157923e+06	8.878665516620094e+06	1.513454128985574e+06	6.370817712742206e+06	2.2333e−11
HHO	2.965762292086701e+06	3.049913063061102e+06	2.794727703840724e+04	3.009015848349671e+06	2.2333e−11
MVO	2.965028069187108e+06	3.104935546744041e+06	4.725450506359614e+04	3.000636615238501e+06	2.2333e−11
WOA	2.964959923759075e+06	3.065800243111563e+06	2.717255105422371e+04	2.981166839528273e+06	2.2333e−11
SSA	2.972872257844985e+06	3.462237622693680e+06	1.208272969624692e+05	3.097109665306138e+06	2.2333e−11
GWO	2.964913731953692e+06	2.965345736335006e+06	1.175047218867738e+02	2.965072393486375e+06	2.2333e−11
PRO	Inf	Inf	Inf	Inf	Inf

The bold numbers in a table are the best result that shown in each row of the table

CEC2019 test functions were used and compared with popular and new algorithms. The results showed that the proposed algorithm had a good ability to solve these test functions. Also, to show the greater efficiency of the proposed algorithm, several real problems used in different fields of study and the results showed that the proposed algorithm performed very well. In contrast, some algorithms were not able to solve these problems.

Simplicity, ease of use, along with its effective and efficient results, can highlight the proposed algorithm as an alternative optimization algorithm for classical methods. Solving other optimization problems in different disciplines is recommended to evaluate the proposed algorithm further. Since the proposed algorithm is a single-objective algorithm, it will be essential to develop a binary and multi-objective version of the proposed algorithm.

Table 19 Results for Himmelblau's function

Algorithm	Optimum cost				
	Min	Max	Std	Avg	*p* value
WHO	**−3.066553867178353e+04**	**−3.066553867178352e+04**	**7.085288006930224e−12**	**−3.066553867178352e+04**	–
PSO	−3.066553867178354e+04	−3.066553867177580e+04	1.877546024177567e−09	−3.066553867178303e+04	9.0073e−06
GA	−3.054556446582087e+04	−3.011477556410298e+04	1.225265548929880e+02	−3.034243078375978e+04	3.1578e−12
AEFA	−3.047410600252781e+04	−2.991796817089555e+04	1.425175808742094e+02	−3.020826289485204e+04	3.1578e−12
FA	−3.066552320039300e+04	−3.047132689213054e+04	39.763041785616402	−3.064579061852828e+04	3.1578e−12
GSA	−3.028984452165243e+04	−2.971549950423253e+04	−2.997502169320204e+04	−2.997502169320204e+04	3.1578e−12
HHO	−3.066255308230259e+04	−3.020647783943505e+04	1.571381640355659e+02	−3.053359534318217e+04	3.1578e−12
MVO	−3.066206253565386e+04	−3.021613453853306e+04	96.807947472564663	−3.057885154733720e+04	3.1578e−12
WOA	−3.059199457650914e+04	−2.952210566024509e+04	2.637846402151887e+02	−3.009266454809931e+04	3.1578e−12
SSA	−3.066174930355314e+04	−3.038304368088696e+04	94.631408854729514	−3.055181785506646e+04	3.1578e−12
GWO	−3.066488133947743e+04	−3.066102540478678e+04	1.100448163605138	−3.066329882004054e+04	3.1578e−12
PRO	−3.124085197910097e+04	−2.978559177761383e+04	2.444437594094266e+02	−3.076985321901556e+04	3.7201e−09

The bold numbers in a table are the best result that shown in each row of the table

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