1. Mathematical model

Reviewing the most recent studies shows, that location-routing optimization by remotely piloted aircraft systems (RPAS) has been modeled from different aspects. This study introduces a new multi-objective model integrating location, routing problem issues as a joint location- routing problem (MLRP). This model considers delivery and pick up demands simultaneously and time window to increase customer satisfaction. In addition, this research takes into account different factors affecting the stability of routes to select the best route with the lowest level of overall risk for shipment of parcels. We try to cover all characteristics, which need to be estimated in this model. It is assumed that in a three echelon supply chain of a delivery pick up dispatching system, a set of k remotely piloted aircraft (RPA) provide services at first, for demand points at the lowest level having a set of P locations. Second, a set of F facility locations at the highest level with a set of D distributors. Finally, for the level of distributors with a set of candidate locations D for construction. Locating at the second echelon occurred simultaneously as the best route is optimized in the whole process of the supply chain. The problem is modeled in the form of a multi-objective mixed-integer linear programming (MILP) model as presented below.

1.1. Objective Function

1.1.1. The function of cost

Delivery to demand points is done by a heterogeneous transport fleet in which RPAs have different velocities, capacities, battery consumption, and purchase costs. Distributors that are used and constructed must be assigned to a facility location. Each RPA is assigned to only one distributor, and the start and end of its route will be from the same distributor. Two periods are defined as time constraints for providing services to demand points. The first constraint is the hard time window. In the hard time window, it is not allowed to provide service to demand point i earlier than time α_i and later than time α_i .The second constraint is the soft time constraint in the period $[Es_i,\ Ls_i]$, meaning that if demand point service i is performed earlier than time Es_i and later than the time Ls_i , demand point service is provided, but the system will be penalized, and the deviation will continue as long as the hard time window is not violated. In that case, no service will be provided at that time. Each demand point has two types of certain demand: the delivery demand based on which the goods are loaded from the distributer and delivered to the demand point and the pickup demand based on which the goods are picked up from the demand point and returned to the distributer.

Distributer centers are divided into two types D_1 and type D_2. D_1 distributers are those where drones can recharge at their location, and D_2 distributers are those where drones cannot recharge at their location. Each distributer is serviced once and only once on one route by one RPA. Each RPA is used in one route only and is assigned to only one distributor.

1.1.2. The function of time

Another constraint defined in the problem is the time constraint for the RPA backhaul to a distributor. For each demand point, the loading operation is done after the delivery of the goods. In this study, RPA velocity has different modes according to different routing scenarios while they are distinctive by the level of air traffic conditions. In a way that, there are a set of U scenarios adjusted with possible routes which end in the set of j nodes. This optimization is accomplished by the second objective (Z2).

In RPA delivery system, air traffic occurred as a result of causes such as:

Air parcels: The cause of the cancellation or delay was due to circumstances within the airline's control (e.g. maintenance or volume problems, uploading/unloading, charging, etc.).

Extreme Weather: Significant meteorological conditions (actual or forecasted) that, in the judgment of the carrier, delays or prevents the operation of a flight such as tornado, blizzard or hurricane.

National Aviation System (NAS): Delays and cancellations attributable to the national aviation system that refer to a broad set of conditions, such as non-extreme weather conditions, airport operations, heavy traffic volume, and air traffic control.

Late-arriving $\mathbf{RPA_S}$: A previous flight with same RPA arrived late, causing the present flight to depart late.

Security: Delays or cancellations caused by re-loading of RPA_S because of security breach, inoperative screening equipment.

Traffic model

1.1.3. The function of risk

To enhance the security and safety of routing plan, this paper develops the third objective function minimizing the total imposed risk during the routes. Since there are a number of risk can be accounted for RPA dispatching system, we need a feature to classify them into specific categories. In this study, we present the methodology for conducting a risk analysis for UAV based on SORA documents. The Specific Operations Risk Assessment (SORA) methodology are being developed by working group 6 (WG6) of the Joint Authorities for the Rulemaking of Unmanned Systems (JARUS). It has been endorsed by the European Aviation Safety Agency (EASA). Then in order to make a meaningful indexes of evaluated risk based on Erkut and Ingolfsson (2005), the risk index for each route was estimated as$\sum_{(i,j)\in A} \breve{p}_{ij} l_i^k$ where $\breve{p}_{ij}$ and $l_i^k$are the probability of an incident occurrence and its consequences, respectively. l_i^k can be defined as the amount of valuables being carried by the drones. The weight of each factor are achieved using the decision maker's opinions, but it should be note that each of mentioned risk factors have different degree of importance. Converting these factors to a single factor is applied utilizing AHP method. Accordingly, the risk of route (S_{ij}^{k}) is calculated based on the instruction stated as follows:

1- Determining effective factors on risk of the route (determined by SORA methodology) and weighting them,

2- Determining weights of each factor using average.

3- Computing risk of each route using sum of weights allocated to route risk factors.

4- Composing a matrix for risk of the routes from node i to node j.

5- Selecting the route with the lowest risk.

The factors must be defined before the optimization of the model, so they are calculated according to the related various features. AHP is a multi-criterion decision-making (MCDM) method for ranking a number of alternatives with respect to their various criteria. The AHP is applicable when the weight of proposed criteria is unknown. According to the AHP methodology, each possible route can be ranked by the risks measured in weights. They are estimated by individual experts' experiences and recorded related researches.

By taking advantage of the proposed model, the risk value can be considerably reduced with only a slight increase in the classical objective function value.

sets

Decision Variables

D:	**A set of distributor candidate locations**
F:	A set of facility location
P:	**A set of demand points**
K:	A set of drones
P1:	A set of demand points where recharging is possible
P2:	A set of demand points where recharging is not possible
U:	A set of different scenarios

Parameters

Parameters	Description

r_i:	The amount of delivery demand of demand points i
Pi:	The amount of pickup demand of demand points i
S_{iu}:	Time to provide service to demand points I for scenario u
Q_k:	Loading capacity of each RPA k
a_i :	The earliest time allowed to provide service to distributor i in the hard time window
b_i:	The latest time allowed to provide service to distributor i in the hard time window
M:	Optional large number
ES_i:	The earliest time allowed to provide service to distributor i in the soft time window
LS_i:	The latest time allowed to provide service to distributor i in the soft time window
W_2:	Cost per unit time deviation from the earliest time allowed in the soft time window
W_3:	Cost per unit time deviation from the latest time allowed in the soft time window
$fix_k^/$:	Fixed cost of using RPA k
C:	Cost of one charging unit
AT:	Minimum amount of charging allowed inside the RPA
Cf_f:	Preparing cost of a unit in facility location f
fix_d:	Cost of constructing distributor candidate location d
DAY:	The length of a working day

cp_{ji}:	RPA battery consumption from node i to node j
dx_{ij}:	The distance between node i and node j
full:	Battery charging capacity
p_u:	The probability of occurrence of scenarios u
V_{ku}:	The velocity of RPA k in event u
cap_d:	The capacity of candidate location of distributor d
$\overline{cap}_f$:	The capacity of facility location f
S_{ij}^k:	The risk of route deriving from node i to node j with RPA k

Decision Variables	
x_{ijkd}:	The variables zero and one. If RPA $k \in K$ belonging to candidate distributor locations d travels from node i to node j, it is equal to one and otherwise zero.
s_{iu}:	The time to start providing service to demand point i in scenario u
L_j:	The weight of the load remaining on the RPA after service to demand point j
Z_d	The variables zero and one. If distributor d is constructed, it is equal to one and otherwise zero
E_{iu}:	The time deviation from the earliest time allowed to provide service to demand point i in the soft time window in scenario u
L_{iu}:	The time deviation from the latest time allowed to provide service to demand point i in the soft time window in scenario u
FC_k	The time to end the route of RPA k
de_d	The center demand of distributor d
Y_{df}	Variables zero and one. If distributor d is assigned to facility location f, it is equal to one and otherwise zero.
A_i:	The amount of battery available on the RPA
LO_k:	The load on RPA k when leaving the distributor

$$Z_1 = min \sum_{k\in K} \sum_{d\in v_d} \sum_{j\in v} \sum_{i\in v} X_{ijkd} . FC_k . dx_{ij} . C \ , + W_2 . \sum_{i\in p} E_i + W_3 . \sum_{i\in p} L_i + \sum_{d\in D} \sum_{k\in K} Z_d . fix_d$$

$$+\sum_{d\in D}\sum_{k\in K}\sum_{i\in p} X_{dikd}.fix_k^{/} + \sum_{d\in D}\sum_{f\in F} Y_{df}.de_d.Cap_f \quad (1)$$

An objective function Z1 that minimizes the total operation cost for the drones transferring, cost of battery consumption along each route, the fixed cost of using the drones, the cost of constructing distributors at candidate locations, the cost of not complying with the soft time window, and the cost of preparing.

$$\boldsymbol{Z_2 = min} \sum_{u\epsilon U} p_u.\sum_{i\epsilon P} s_{iu} + \lambda \sum_{u\epsilon U} p_u \left(\sum_{i\epsilon P} s_{iu} - \sum_{u\epsilon U} p_u \sum_{i\epsilon P} s_{iu}\right)^2$$

In the second objective function, the service time is minimized.

$$\boldsymbol{Z3 = Min} \sum_{\boldsymbol{i,j\ i\in P}\cup \boldsymbol{D}} \sum_{d\in D}\sum_{k\in K}\sum_{i\in P\cup D} s_{ij}^k X_{ijkd} \quad (3)$$

The third objective function is modeled to minimize risk index, it is measured by the sum of the imposed routing risk the weight of which is multiplied in the weights of its included components(factors) for each node i to j.

Subject to:

1. $\sum_{d\in D}\sum_{k\in K}\sum_{i\in P\cup D} X_{ijkd} = 1 \quad ,\forall j\in P$
2. $\sum_{j\in P\cup D} X_{djkd} = \sum_{i\in P\cup D} X_{jdkd} \quad ,\forall d\in D, k\in K$
3. $\sum_{i\in D\cup P} X_{ijkd} = \sum_{i\in D\cup P} X_{jikd} \quad ,\forall d\in D, k\in K, j\in P$
4. $\sum_{d\in D}\sum_{i\in v} X_{dikd} \le 1 \quad ,\forall k\in K$
5. $\sum_{i\in D, i\ne d}\sum_{j\in P} X_{ijkd} = 0 \quad ,\forall d\in D, k\in K$
6. $\sum_{i\in P}\sum_{j\in D, j\ne d} X_{ijkd} = 0 \quad ,\forall d\in D, k\in K$

7 X_{iikd}=0 $\quad ,\forall d\in D, k\in K, i\in D\cup P$

8. $S_{iu} + \frac{dx_{ij}}{V_{ku}} - M.\left(1 - X_{ijkd}\right) \le S_{ju}$ $\quad ,\forall d \in D, k \in K, i \in D \cup P, j \in P, u \in U$

9. $S_{iu} + \frac{dx_{ij}}{V_{ku}} + M.\left(1 - X_{ijkd}\right) \ge S_{ju}$ $\quad ,\forall d \in D, k \in K, i \in D \cup P, j \in P, \mathrm{u} \in U$

10. $S_{du} = 0$ $\quad ,\forall d \in D, \mathrm{u} \in U$

11. $a_i \le S_{iu} \le b_i$ $\quad ,\forall i \in P, \mathrm{u} \in U$

12. $LO_k = \sum_{d\in D} \sum_{i\in v} \sum_{j\in P} r_j \times X_{ijkd}$ $\quad ,\forall k \in K$

13. $LO_k \le Q_k$ $\quad ,\forall k \in K$

14. $L_j \ge LO_k - r_j + p_j - M.\left(1 - X_{djkd}\right)$ $\quad ,\forall d \in D, k \in K, j \in P$

15. $L_j \le LO_k - r_j + p_j + M.\left(1 - X_{djkd}\right)$ $\quad ,\forall d \in D, k \in K, j \in P$

16. $L_j \ge L_i - r_j + p_j - M.(1 - \sum_{d\in v_d} \sum_{k\in K} X_{ijkd})$ $\quad ,\forall j \in P ,\forall_i \in D$

17. $L_j \le L_i - r_j + p_j + M.(1 - \sum_{d\in v_d} \sum_{k\in K} X_{ijkd})$ $\quad ,\forall j \in P, \forall_i \in D$

18. $L_j.(\sum_{d\in D} \sum_{i\in P\cup D} X_{ijkd}) \le Q_k$ $\quad ,\forall j \in P$

19. $\mathrm{E_{iu}} \ge (\mathrm{ES_i} - \mathrm{S_{iu}})$ $\quad ,\forall \mathrm{i} \in P , \mathrm{u} \in U$

20 $L_{iu} \ge (S_{iu} - LS_i),$ $\quad ,\forall \mathrm{i} \in \mathrm{P} , \mathrm{u} \in U$

21 $\mathrm{A_i} = \mathrm{full}$ $\quad , \forall i \in P_1$

22 $\mathrm{A_i} \le A_j - DX_{ji}.CP_{ji} + M.\left(1 - X_{jikd}\right)$, $\quad \forall i \in P_2, j \in P \cup D, k \in K, d \in D$

23 $\mathrm{A_i} \ge \mathrm{AT}$ $\quad , \forall \mathrm{i} \in \mathrm{P}$

24 $Z_d.M \ge \sum_{k\in K} \sum_{i\in P} X_{dikd}$ $\quad ,\forall d \in D$

25. $M.\sum_{f\in F} Y_{df} \ge Z_d$ $\quad ,\forall d \in D$

26. $de_d = \sum_{k\in K} \sum_{i\in P\cup D} \sum_{j\in P} de_d \times X_{ijkd}$ $\quad ,\forall d \in D$

27. $de_d \le cap_d$ $\quad ,\forall d \in D$

28. $\sum_{f\in F} \sum_{d\in D} Y_{df}.de_d \le \overline{cap}_f$ $\quad ,\forall f \in F$

29. $FC_k \ge S_{iu} + \frac{dx_{id}}{V_{ku}} - M.(1 - X_{idkd})$ $\quad ,\forall k \in K, i \in P, d \in D, u \in U$

30 $\sum_{k\in K} \sum_{i\in R\cup D} \sum_{j\in R} S_i \times X_{ijkd} \le DAY$ $\quad ,\forall k \in K,$

31. $X_{ijkd}, Z_d, Y_{df} = 0 \text{ or } 1$

32. $L_j, S_{iu}, LO_{dk} \geq 0$

33. $s_{ij}^k \leq \mu \sum_{i \epsilon P \cup D} X_{jikd}$, $\forall j \in P, k \in K$

34. $s_{ij}^k \leq \mu \sum_{i \epsilon P \cup D} X_{ijkd}$, $\forall j \in P, k \in K$

35. $s_{ij}^k = 0$, $\forall j \in D, k \in K$

Constraint (1) ensures that a RPA enters each demand point and meets the point's demand.

Constraint (2) ensures that any RPA leaving the distributor returns to the same distributor at the end of the route.

Constraint (3) ensures that any RPA that enters a node to provide service to any demand points exits that node.

Constraint (4) ensures that each RPA is used in only one distributor.

Constraints (5 and 6) ensure that if a RPA leaves a distributor, that RPA belongs only to that distributor.

Constraint (7) ensures that there is no edge from each node to itself.

Constraints (8 and 9) calculate the start time of service to each demand point.

Constraint (10) ensures that the start time of the RPA is zero.

Constraint (11) ensures compliance with the hard time window limit.

Constraint (12) calculates the amount of initial loading of the RPA.

Constraint (13) examines the capacity constraint of the RPA for the initial loading.

Constraints (14 and 15) calculate the weight of the load on the RPA after leaving the first demand point along the route.

Constraints (16 and 17) calculate the weight of the load on the RPA after leaving other demand points

Constraint (18) examines the capacity constraint of the RPA for the weight of the load on the RPA along the route.

Constraint (19) calculates the deviation from the soft time window for demand points.

Constraints (19 and 20) calculate the soft time window deviation for demand points.

Constraint (21) indicates that at the end of the service to demand points where charging is possible, the amount of battery in the RPA capacity is full.

Constraint (22) calculates the amount of battery in the RPA battery capacity at the end of service to demand points where recharging is not possible.

Constraint (23) ensures that the RPA battery constraint is observed.

Constraint (24) identifies constructed distributors.

Constraint (25) ensures that each distributor constructed is assigned to one facility location.

Constraint (26) calculates the demand of each distributor.

Constraint (27) examines the demand constraint of each distributor.

Constraint (28) examines the demand constraint of each factory.

Constraint (29) calculates the travel time for each RPA.

Constraint (30) examines the time constraint of the travel length in a day.

Constraint (31) specifies the variables zero and one.

Constraint (32) specifies variables greater than or equal to zero.

Constraint (34) ensures that the imposed risk for route i to j is lower than M denotes a large positive constant.

Constraint (35) ensures that the imposed risk for route j to i is lower than M denotes a large positive constant.

Constraint (36) ensures that risk of routes ending in facility locations are zero

1.2.Assumptions

-The drones make one-to-many delivery trips at the first echelon (from the facility location to the distribution centers) and many to many trips at the second echelon (from distribution centers to demand points and back) until the battery is exhausted.

-We do not consider one-to-many vehicle routing type trips, which is consistent with the initial applications of RPA deliveries by private companies.

-We also do not consider battery recharging during the planning period and assume that the RPA battery is recharged between planning periods.

-The length of the planning period is shorter (6 hours to a day or 2 days) compared with the planning period for a typical facility location problem.

-Another important assumption for the development of the mathematical model is related to the first m actions. They are dummy event locations used to represent the UAVs initial positions. Thus, no time windows are associated to these events, but they simply are born at the scenario starting time instant and remain active for all durations of the scenario itself.

-we refer to the RPA as any aircraft capable of moving autonomously at varied and heterogeneous velocity, capacity, battery consumption, purchasing cost.

- Each RPA can serve more than one demand point per dispatch as long as its flight range and load carrying capacity are not violated.

- RPAs can return to same distribution centers along the route.

-Multiple RPAs can be dispatched simultaneously from any distribution centers, which allows the use of a swarm of RPAs to enhance the overall productivity of the system.

-We assume that applied RPAs have the battery capacity to cover the longest planning period, and before per dispatch, operators ensure that drones have enough battery to the end of the route.

- The whole of the distribution centers have equipped with recharging stations

- Customers are served only by RPAs.

- RPAs can be dispatched and collected several times from the same distribution centers.

- RPA batteries are replaced with fully charged batteries each time.

- Packages are loaded and unloaded from the RPAs once the RPAs deliver to distribution centers or demand points

- After battery replacement and package loading/unloading, the RPA can be dispatched again to serve a new set of demand points. The process is repeated until all demand points in the service area have been reached.